Saarland University
FACULTY 6.1 – MATH
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Exercises for the lecture topology

Winter term 2019/2020

Sheet 2 Deadline: 10.29.2019

Remark: Please deliver your solution of the exercise sheet on Tuesday just before the lecture.

Exercise 5 (2+2+1=5 Points)

Let (X, d) be a metric space and let $a \in X$ and $r > 0$ be given.

(a) Show the inclusions $\partial B_r(a) \subset \{x \in X;\ d(x, a) = r\}$ and $\overline{B_r(a)} \subset \overline{B}_r(a)$.

(b) Suppose that $(X, \|\cdot\|)$ is a normed space and that $d : X \times X \to \mathbb{R}$, $d(x, y) = \|x - y\|$ is the induced metric. Show that we have in this case the identities $\partial B_r(a) = \{x \in X;\ d(x, a) = r\}$ and $\overline{B_r(a)} = \overline{B}_r(a)$.

(c) Show that in general the identity $\partial B_r(a) = \{x \in X;\ d(x, a) = r\}$ need not be true.

Exercise 6 (4x0.5+0.5+1+0.5=4 Points)

(a) Show that there are subsets $A, B \subset \mathbb{R}$ for which the following identities are false:

(i) $\overline{A \cap B} = \overline{A} \cap \overline{B}$,

(ii) $\operatorname{Int}(A \cup B) = \operatorname{Int}(A) \cup \operatorname{Int}(B)$,

(iii) $\partial(A \cup B) = (\partial A) \cup (\partial B)$,

(iv) $\partial A = \partial \overline{A}$.

(b) Let (X, d) be a metric space and $A \subset X$. Decide whether the following identities are true or false:

(i) $\operatorname{Int}(\overline{A}) = A$,

(ii) $\operatorname{Int}(\overline{\operatorname{Int}(\overline{A})}) = \operatorname{Int}(\overline{A})$,

(iii) $\overline{\operatorname{Int}(A)} = A$,

A family $(A_i)_{i \in I}$ of subsets of a metric space X is called ***locally finite*** *if for each point $x \in X$ there is a positive real number $\varepsilon > 0$ such that the set $\{i \in I;\ A_i \cap B_\varepsilon(x) \neq \emptyset\}$ is finite.*

Exercise 7 (4 Points)

Let $(A_i)_{i \in I}$ be a locally finite family of subsets of a metric space X. Show that $\bigcup_{i \in I} \overline{A_i} \subset X$ is closed.

(please turn over)

Exercise 8 (2+2=4 Points)

Let (X_1, d_1), (X_2, d_2) be two metric spaces and let $f : X_1 \to X_2$ be a surjective continuous map such that $d_1(x, y) \leq d_2(f(x), f(y))$ for all $x, y \in X_1$. Prove or disprove the following assertions:

(a) If X_1 is complete, then X_2 is complete.

(b) If X_2 is complete, then X_1 is complete.

(*Hint : The result of Exercise 3 might help.*)

You can also find the exercise sheets on our homepage:

https://www.math.uni-sb.de/ag/eschmeier/lehre/WS1920/top/index.html