Saarland University
FACULTY 6.1 – MATH
Prof. Dr. Jörg Eschmeier
M.Sc. Manuel Kany

Exercises for the lecture topology

Winter term 2019/2020

Sheet 5 Deadline: 11.19.2019

Exercise 17 (4 Points)

Let $K : [0,1] \times [0,1] \to \mathbb{R}$ be a continuous function with $|K(x,y)| < 1$ for all $x, y \in [0,1]$. Show that there is a unique continuous function $f : [0,1] \to \mathbb{R}$ such that the equation

$$f(x) + \int_0^1 K(x,y) f(y)\, dy = e^{x^2}$$

holds for all $x \in [0,1]$.

Exercise 18 (4 Points)

Let $X \neq \emptyset$ be a set. Decide whether the following collections of sets $t_i \subset \mathcal{P}(X)$ $(i = 1,2,3)$ define topologies on X and under which circumstances they are Hausdorff spaces:

$$\begin{array}{lcl} U \in t_1 & \Leftrightarrow & U = \emptyset \text{ or } X \setminus U \text{ is finite} \\ U \in t_2 & \Leftrightarrow & U = \emptyset \text{ or } X \setminus U \text{ is countable} \\ U \in t_3 & \Leftrightarrow & U = \emptyset \text{ or } U = X \text{ or } X \setminus U \text{ is not finite.} \end{array}$$

Exercise 19 (1+1+2=4 Points)

Let (X,t) be a topological space and let $\emptyset \neq Y \subset X, A \subset Y, K \subset X$ be subsets. Show the following assertions:

(a) A is closed in $(Y, t|_Y)$ if and only if there is a closed set $F \subset X$ with $A = F \cap Y$.

(b) A is compact in $(Y, t|_Y)$ if and only if A is compact in X.

(c) K is compact if and only if, for every family $(F_i)_{i \in I}$ of closed sets F_i in $(K, t|_K)$ which has the finite intersection property the intersection $\bigcap_{i \in I} F_i$ is nonempty.

Exercise 20 (4 Points)

Let X, Y be topological spaces and let $X = A \cup B$ be the union of closed (or of open) subsets. Furthermore let $f_A : A \to Y$ and $f_B : B \to Y$ be continuous maps (with respect to the relative topology on X) such that $f_A = f_B$ on $A \cap B$. Show that the map $f : X \to Y$ defined by

$$f(x) = \begin{cases} f_A(x), & \text{for } x \in A \\ f_B(x), & \text{for } x \in B \end{cases}$$

is continuous.

(please turn over)

You can also find the exercise sheets on our homepage:

https://www.math.uni-sb.de/ag/eschmeier/lehre/WS1920/top/index.html