Saarland University
FACULTY 6.1 – MATH
Prof. Dr. Jörg Eschmeier
M.Sc. Manuel Kany

Exercises for the lecture topology

Winter term 2019/2020

Sheet 8 Deadline: 12.10.2019

Exercise 29 (1+2+1=4 Points)

Let $X \neq \emptyset$ be a non-empty set. Show that:

(a) If t and t' are topologies on X, then $t = t'$ if and only if the equivalence

$$\lim_{\alpha} x_\alpha = x \text{ in } (X, t) \iff \lim_{\alpha} x_\alpha = x \text{ in } (X, t')$$

holds for each net $(x_\alpha)_{\alpha \in A}$ in X and each $x \in X$.

(b) The assertion of part (a) does not hold in general if nets are replaced by sequences. (*Hint : Consider* $\mathbb{R}$ *equipped with the topology* $t = t_2$ *from Exercise 18 and the discrete topology* $t' = \mathcal{P}(\mathbb{R})$).

(c) There are sequentially continuous maps which are not continuous.

Exercise 30 (1+1+2=4 Points)

Let (X_i, t_i) $(i \in I)$ be topological spaces. Equip $X = \prod_{i \in I} X_i$ with its product topology t. Let $A = \prod_{i \in I} A_i$ be the cartesian product of subsets $A_i \subset X_i$. Show that

(a) If each $A_i \subset X_i$ $(i \in I)$ is a closed subset, then also $A \subset X$ is closed.

(b) $\overline{A} = \prod\limits_{i \in I} \overline{A_i}$.

(c) $\operatorname{Int}(A) \subset \prod\limits_{i \in I} \operatorname{Int}(A_i)$. Decide whether equality holds here.

A mapping $f : X_1 \to X_2$ *between topological spaces is called open (closed) if* $f(M) \subset X_2$ *is open (closed) for every set* $M \subset X_1$ *that is open (closed).*

(please turn over)

Exercise 31 **(2+2=4 Points)**

Let (X_i, t_i) $(i \in I)$ be topological spaces and let t be the product topology on $X = \prod_{i \in I} X_i$. Show that:

(a) (X, t) is Hausdorff if and only if each of the spaces (X_i, t_i) $(i \in I)$ is a Hausdorff space.

(b) The projections

$$\pi_j : X \to X_j, \ (x_i)_{i \in I} \mapsto x_j \ (j \in I)$$

are open but in general not closed.

Exercise 32 **(4 Points)**

Let (X_i, t_i) $(i \in I)$ be separable topological spaces. Show that $X = \prod_{i \in I} X_i$ equipped with the product topology is a separable topological space.

You can also find the exercise sheets on our homepage:

https://www.math.uni-sb.de/ag/eschmeier/lehre/WS1920/top/index.html