Saarland University
FACULTY 6.1 – MATH
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Exercises for the lecture topology

Winter term 2019/2020

Sheet 9 Deadline: 12.17.2019

Exercise 33 (4 Points)

Let I be an uncountable index set and let (X_i, t_i) $(i \in I)$ be given topological spaces such that $t_i \neq \{\emptyset, X_i\}$ for each $i \in I$. Show that the product topology t on $X = \prod_{i \in I} X_i$ is not first countable. (*Hint : Choose open sets $U_i \subsetneq X_i$ and elements $x_i \in U_i$ and show by contradiction that $x = (x_i)_{i \in I}$ cannot possess a countable neighbourhood base.*)

Exercise 34 (4 Points)

Equip $\{0, 1\}$ with its discrete topology and $X = \prod_{A \subset \mathbb{N}} \{0, 1\} (= \{0, 1\}^{\mathcal{P}(\mathbb{N})})$ with its product topology. Show that X is compact, but not sequentially compact. (*Hint : Consider the sequence $(x^{(n)})_{n \in \mathbb{N}}$ in X defined by: $x_A^{(n)} = 1 \quad :\Longleftrightarrow \quad n \in A$ and the number of elements in $\{k \in A;\ k < n\}$ is even.*)

Exercise 35 (4 Points)

Let X, Y be topological spaces, let $\emptyset \neq K_1 \subset X, \emptyset \neq K_2 \subset Y$ be compact sets and let $W \subset X \times Y$ be open in the product topology such that $K_1 \times K_2 \subset W$. Show that there are open sets $U \subset X$, $V \subset Y$ with $K_1 \subset U$, $K_2 \subset V$ and $U \times V \subset W$.

Exercise 36 (4 Points)

Let X be a topological space and let K be a compact Hausdorff space. Show that a mapping $f : X \to K$ is continuous if and only if its graph $G_f = \{(x, f(x));\ x \in X\} \subset X \times K$ is closed with respect to the product topology. (*Hint : Argue for "$\Leftarrow$ "by contradiction as in the proof of theorem 6.7.*)

You can also find the exercise sheets on our homepage:

https://www.math.uni-sb.de/ag/eschmeier/lehre/WS1920/top/index.html