SAARLAND UNIVERSITY
DEPARTMENT OF MATHEMATICS
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Exercises for the lecture Functional Analysis I
Winter term 2020/2021

Sheet 9 To be submitted until: Thursday, 21.01.2021, before the lecture

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Exercise 37 (2+2=4 Points)

Let X, Y, Z be normed spaces and let $T\colon X \times Y \to Z$ be a bilinear map. Show:

(a) T is continuous if and only if there is a constant $c > 0$ such that $\|T(x,y)\| \leq c\|x\|\|y\|$ for all $x \in X$ and $y \in Y$.

(b) If X, Y, Z are Banach spaces and if the mappings $T(x,\cdot)$ $(x \in X)$ and $T(\cdot, y)$ $(y \in Y)$ are continuous, then T is continuous.
(*Hint : Consider the mappings* $T(x,\cdot)$ *with* $x \in B_X$.)

Exercise 38 (2+2=4 Points)

Let $T\colon X \to Y$ be a compact operator between Banach spaces X and Y. Show:

(a) If T is surjective, then $\dim(Y) < \infty$.

(b) If $M \subset Y$ is a closed subspace with $M \subset TX$, then $\dim(M) < \infty$.

Exercise 39 (2+3=5 Points)

(a) Let A be a unital complex Banach algebra. For $a \in A$, let $L_a\colon A \to A$, $a \mapsto ax$, be the operator induced by left multiplication with a. Show that $\sigma(a) = \sigma(L_a)$.

(b) Determine all functions $\varphi \in C[0,1]$ for which the induced multiplication operator $M_\varphi\colon C[0,1] \to C[0,1]$, $f \to \varphi f$, is compact.

(please turn over)

Exercise 40 (4 Points)

Let X, Y be Banach spaces and $T \in \mathcal{L}(X, Y)$. Show that

$$T \in \mathrm{Fred}(X, Y) \Leftrightarrow T' \in \mathrm{Fred}(Y', X')$$

and in this case $\operatorname{ind} T' = \operatorname{ind} T$.
(*Hint : Use Theorem 6.15 and Corollary 8.8.*)

You may submit the solutions for the exercise sheets in groups up to three participants, belonging to the same tutorial group. " But please avoid to meet in person. To be admitted to the exam, you need 50 % of the points achievable in the homework assignments. Homework marked with a star allows you to achieve additional points. ***This time all Exercises will be corrected!***

You can also find the exercise sheets on our homepage:

http://www.math.uni-sb.de/ag/eschmeier/lehre