Universität des Saarlandes
Fachrichtung 6.1 – Mathematik
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PDE and Boundary-Value Problems (Winter Term 2013/2014)
Assignment H4 - Homework

Problem 4.1 (Laplace Transform - 6 Points)

Solve the problem

$$\begin{aligned} \text{PDE:}\quad & u_t = u_{xx}, && 0 < x < \infty, \quad 0 < t < \infty \\ \text{BC:}\quad & u(0,t) = \sin(t), && 0 < t < \infty \\ \text{IC:}\quad & u(x,0) = 0, && 0 \leqslant x < t\infty \end{aligned}$$

by means of the Laplace transform (transform t). What is the physical interpretation of this problem?

Problem 4.2 (Laplace Transform and Maple - 8 Points)

We seek the temperature distribution $u(x,t)$ in a thin rod over the interval $0 < x < \infty$ whose lateral surface is insulated. The left end of the rod is constarined at a varying temperature $q(t) = \dfrac{1}{\sqrt{t}}$, and the rod has an initial temperature distribution $u(x,0) = 0$. There is no heat source in the system and the termal diffusivity of the rod is $k = 1/50$.

Using Maple solve the problem by means of the Laplace transform (transform t). Show the animation of the spatial-time temperature distribution $u(x,t)$ in the rod for $0 \leqslant t \leqslant 5$.

(*Hinweis:* use the Maple commands **laplace** and **invlaplace** for the Laplace transform and the inverse Laplace transform, respectively.)

Problem 4.3 (Solving IBVP - 6 Points)

Suppose we have a metal rod (laterally insulated) and we supply an *initial impulse of heat* at the right-hand side (the left-hand side is fixed at zero). Suppose the initial temperature of the rod is zero (some reference temperature) and the temperature at the midpoint $x = 0.5$ is measured at various values of time, so that we have the following table:

Values of Time	**Midpoint Temperature**
t_1	w_1
$t_2 = 2t_1$	w_2
$t_3 = 3t_1$	w_3
$\vdots$	$\vdots$
$t_n = nt_1$	w_n

Using this data, how could we approximate the temperature responce at the points $u(0.5, t_n)$ due to the BCs

(a) $u(1,t) = \sin(t)$

(b) $u(x,t) = f(t)$ (arbitrary $f(t)$)?

Problem 4.4 (Duhamel's Principle - 4 Points)

Using Duhamel's principle, what is the solution of the IBVP

$$\text{PDE:} \quad u_t = \alpha^2 u_{xx}, \qquad 0 < x < 1, \quad 0 < t < \infty$$

$$\text{BCs:} \quad \begin{cases} u(0,t) = 0 \\ u(1,t) = \sin(t) \end{cases}, \qquad 0 < t < \infty$$

$$\text{IC:} \quad u(x,0) = 0, \qquad 0 \leqslant x \leqslant 1$$

Deadline for submission: Wednesday, December 11, 10am