Universität des Saarlandes
Fachrichtung 6.1 – Mathematik
PD Dr. Darya Apushkinskaya

PDE and Boundary-Value Problems (Winter Term 2013/2014)
Assignment H6 - Homework

Problem 6.1 (Wave Problem in Polar Coordinates - 8+6=14 Points)

a) Solve the wave problem in polar coordinates

$$\text{PDE:}\quad u_{tt} = \Delta u, \quad 0 < r < 1, \quad 0 \leqslant \theta < 2\pi, \qquad 0 < t < \infty$$

$$\text{BC:}\quad u(1,\theta,t) = 0, \qquad 0 < t < \infty$$

$$\text{ICs:}\quad \begin{cases} u(r,\theta,0) = J_0(2.4r) - 0.5J_0(8.65r) + 0.25J_0(14.93r), \\ u_t(r,\theta,0) = 0, \end{cases} \qquad 0 \leqslant r \leqslant 1$$

by using the method of separation of variables.

b) Sketch the nodal lines for fundamental vibrations U_{34}, U_{33}, and U_{24}.

Problem 6.2 (2-Dimensional Wave Equation - 5 Points)

Let $u(x,y,t)$ be the solution of the two-dimensional wave equation

$$u_{tt} = c^2(u_{xx} + u_{yy}), \quad (x,y) \in \mathbb{R}^2,\ 0 < t, \infty$$

with initial conditions

$$u(x,y,0) \equiv 0$$
$$u_t(x,y,0) = \begin{cases} 1, & \text{if } x^2 + y^2 \leqslant 4, \\ 0, & \text{otherwise.} \end{cases}$$

Find $u(0,0,t)$ for all $t > 0$ and determine the behavior of $u(0,0,t)$ when $t \to \infty$.

Problem 6.3 (The Laplacian - 4 Points)

Transform the three-dimensional Laplacian into cylindrical coordinates and spherical coordinates, respectively.

Problem 6.4 (The Neumann problem - 4 Points)

Does the following Neumann problem have a solution inside the circle:

$$\text{PDE:} \quad \Delta u = 0, \quad 0 < r < 1, \quad 0 \leqslant \theta < 2\pi$$

$$\text{BC:} \quad \tfrac{\partial u}{\partial r} = \sin^2 \theta, \quad 0 \leqslant \theta < 2\pi$$

Deadline for submission: Friday, January 24, 10am