UNIVERSITÄT DES SAARLANDES
DEPARTMENT 6.1 – MATHEMATICS
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Exercises for the Lecture
Differential Geometry
Summer Term 2020

Sheet 11 **Submission:** /

Resources: Up to Lesson 20; Chapters 1–2 in [Fuc08]; Chapters 1–3 in [Car16]

Exercise 1.

(i) Show that the parametrization of the elliptic paraboloid
$$X\colon \mathbb{R}^2 \to \mathbb{R}^3,\ (u,v) \mapsto (u, v, u^2 + v^2)$$
has no asymptotic curves.

(ii) Determine the asymptotic curves of the following parametrization of the hyperbolic paraboloid
$$X\colon \mathbb{R}^2 \to \mathbb{R}^3,\ (u,v) \mapsto (u, v, u^2 - v^2).$$

Exercise 2.
(*See Exercise 2 in Section 3-3 in [Car16].*)
Let $a, b > 0$. Consider the *helicloid*
$$X\colon \mathbb{R}^2 \to \mathbb{R}^3,\ (u,v) \mapsto (av\cos(u), av\sin(u), bu).$$

(i) Show that X is a ruled surface. Are the generators asymptotic curves?

(ii) Determine the curvature lines of the surface for $a = b = 1$.
(*Hint: Use* $\widetilde{\omega_2} = \operatorname{arsinh}(\omega_2)$.)

(iii) Show that X is a minimal surface.

Exercise 3.
(*See Exercise 6 in Section 3-3 in [Car16].*)

(i) Let the unit sphere be parameterized by
$$X\colon (0, 2\pi) \times (0, \pi) \to \mathbb{R}^3,\ (u,v) \mapsto (\cos(u)\sin(v), \sin(u)\sin(v), \cos(v)).$$
Calculate the geodesic curvature of all circles of latitude and longitude (u resp. v coordinate lines).

(ii) The *pseudo sphere* is the following regular parameterized rotation surface
$$P^2\colon \mathbb{R} \setminus \{0\} \times \mathbb{R} \to \mathbb{R}^3,\ (u,v) \mapsto \left(\frac{\cos(v)}{\cosh(u)}, \frac{\sin(v)}{\cosh(u)}, u - \tanh(u)\right).$$
Show that the pseudo sphere has constant negative Gauß curvature.

Exercise 4.
Let $\Omega \subset \mathbb{R}^2$ be a domain and let $X\colon \Omega \to \mathbb{R}^3$ be a regular parametrization of a surface. Show that the following statements are equivalent:

(i) $H \equiv K \equiv 0$ on Ω.

(ii) $X(\Omega)$ is a subset of a plane.

(please turn the page)

References

[Car16] Manfredo P. do Carmo. *Differential geometry of curves & surfaces*. Revised & updated second edition. Dover Publications, Inc., Mineola, NY, 2016.

[Fuc08] Martin Fuchs. *Vorlesungsskript zur Differentialgeometrie*. 2008.