> $y = C\cosh\left(\frac{x}{C}\right), \quad y(-1) = y(1) = 2,$
$natural\ catenary(without\ length\ condition)$

> *First, we calculate the value of C for this natural catenary*

> $y(1) = 2 \Leftrightarrow C\cosh\left(\frac{1}{C}\right) = 2 \Leftrightarrow \cosh\left(\frac{1}{C}\right) = \frac{2}{C}$

> $eq := \cosh(x) = 2 \cdot x$

$$eq := \cosh(x) = 2\,x \tag{1}$$

> $solve(eq, x)$

$$RootOf\left(4\,\_Z\,e^{-\_Z} - \left(e^{-\_Z}\right)^2 - 1\right) \tag{2}$$

> $evalf(\%)$

$$0.5893877635 \tag{3}$$

> $eval\left(\frac{1}{x}, x = 0.5893877635\right)$

$$1.696675876 \tag{4}$$

> $y = 1.696675876 \cdot \cosh(0.5893877635 \cdot x)$ - *natural catenary*

>

> $y = C\cosh\left(\frac{x}{C}\right) - \lambda, \quad L = 2 \cdot C \cdot \sinh\left(\frac{x}{C}\right), \quad y(-1) = y(1) = 2$

>

> $L = 2.05$

>

> $eq := \sinh(x) = \frac{2.05}{2} \cdot x$

$$eq := \sinh(x) = 1.025000000\,x \tag{5}$$

> $solve(eq, x)$

$$0.3858596708,\ -0.3858596708,\ 0. \tag{6}$$

> $eval\left(\frac{1}{x}, x = 0.3858596708\right)$

$$2.591615750 \tag{7}$$

> $y(1) = 2 \;\Rightarrow\; \lambda = C \cdot \cosh\left(\frac{1}{C}\right) - 2$

>

> $eval(2.591615750 \cdot \cosh(0.3858596708 \cdot x) - 2, x = 1)$

$$0.786951238 \tag{8}$$

> $y = 2.591615750 \cdot \cosh(0.3858596708 \cdot x) - 0.786951238$ *is a catenary solution for* $L = 2.05$

>

> $L = 2.9$

>

> $eq := \sinh(x) = \frac{2.9}{2} \cdot x$

$$eq := \sinh(x) = 1.450000000\, x \tag{9}$$

> $solve(eq, x)$

$$1.547908780,\ -1.547908780,\ 0. \tag{10}$$

> $eval\left(\frac{1}{x}, x = 1.547908780\right)$

$$0.6460329012 \tag{11}$$

> $eval(0.6460329012 \cdot \cosh(1.547908780 \cdot x) - 2, x = 1)$

$$-0.412593780 \tag{12}$$

> $y = 0.6460329012 \cdot \cosh(1.547908780 \cdot x) + 0.412593780$ *is a catenary solution for* $L = 2.9$

>

> $L = 5$

>

> $eq := \sinh(x) = 2.5 \cdot x$

$$eq := \sinh(x) = 2.5\, x \tag{13}$$

> $solve(eq, x)$

$$-2.552654724,\ 2.552654724,\ 0. \tag{14}$$

> $eval\left(\frac{1}{x},\ x = 2.552654724\right)$

$$0.3917490253 \tag{15}$$

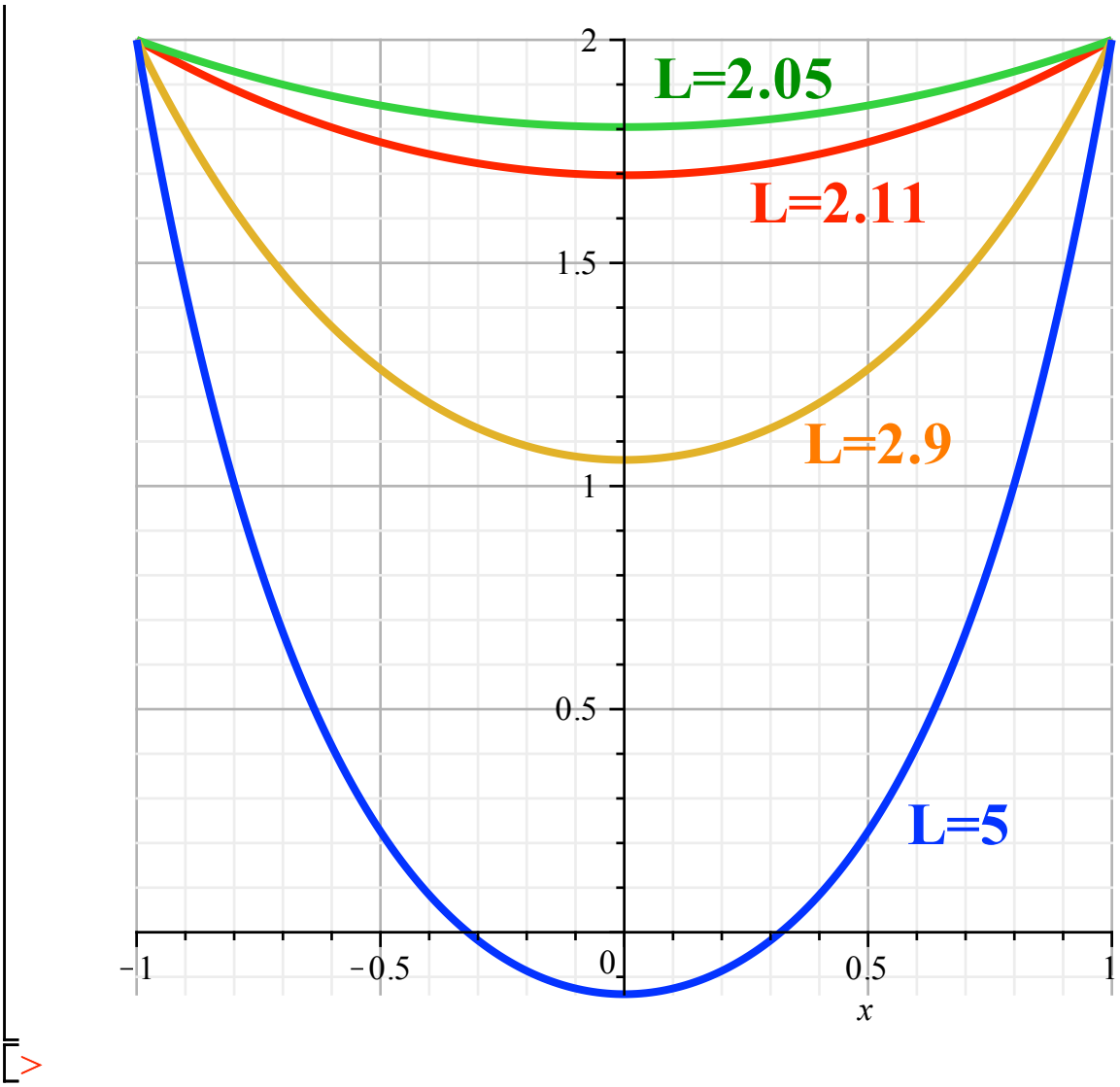
> $eval(0.3917490253 \cdot \cosh(2.552654724 \cdot x) - 2,\ x = 1)$

$$0.530507321 \tag{16}$$

> $y = 0.3917490253 \cdot \cosh(2.552654724 \cdot x) - 0.530507321$ *is a catenary solution for* $L = 5$

>

> $plot(\{1.696675876 \cdot \cosh(0.5893877635 \cdot x),\ 2.591615750 \cdot \cosh(0.3858596708 \cdot x) - 0.786951238,\ 0.6460329012 \cdot \cosh(1.547908780 \cdot x) + 0.412593780,\ 0.3917490253 \cdot \cosh(2.552654724 \cdot x) - 0.530507321\},\ x = -1..1,\ thickness = 3)$


L=2.05
L=2.11
L=2.9
L=5
2
1.5
1
0.5
0
−1
−0.5
0.5
1
x

>