Universität des Saarlandes
Fachrichtung 6.1 – Mathematik
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UNIVERSITÄT DES SAARLANDES

Calculus of Variations (Summer Term 2014)
Assignment H1 - Homework

Problem 1.1 (5+5=10 Points)

We seek to minimize the integral

$$J[y] = \int_0^1 \sqrt{1 + (y')^2} dx.$$

a) Find the minimizing function $\overline{y}(x)$ among those curves satisfying $y(0) = 0$ and $y(1) = 1$. What is your interpretation of your answer? What is the value of $J[\overline{y}]$? What is the meaning of $J[\overline{y}]$?

b) Find the minimizing function $\hat{y}$ among the all curves with the ends lying on the vertical lines $x = 0$ and $x = 1$. Evaluate $J[\hat{y}]$. Compare the obtained results with the answers from item a).

Problem 1.2 (7 Points)

Show that, if y satisfies the Euler-Lagrange equation associated with the integral

$$J[y] = \int_a^b (p^2 y'^2 + q^2 y^2) dx,$$

where $p(x)$ and $q(x)$ are known functions, then $J[y]$ has the value $(p^2 y y')\big|_a^b$.

Problem 1.3 (5 Points)

Derive the differential equation satisfied by the four-times-differentiable function $y(x)$ which minimizes the integral

$$J[y] = \int_a^b F(x, y, y', y'') dx$$

under the condition that both y and y' are prescribed at a and b.

Problem 1.4 (5 Points)

Find an upper bound for the minimum of the functional

$$J[y] = \int_0^1 y^2 \left(y'\right)^2 dx,$$

subject to $y(0) = 0$ and $y(1) = 1$ using the trial functions

$$y_\varepsilon(x) = x^\varepsilon,$$

with $\varepsilon > 1/4$. Justify your argument.

Deadline for submission: Wednesday, May 07, 12 am