**Universität des Saarlandes**
**Fachrichtung 6.1 – Mathematik**
PD Dr. Darya Apushkinskaya

## Calculus of Variations (Summer Term 2014)
## Assignment H3 - Homework

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### Problem 3.1 (4+4=8 Points)

Find the form of extremals of the following functionals

a) $$J_1[y,z] = \int\limits_{x_0}^{x_1} \left(2yz - 2y^2 + y'^2 - z'^2\right) dx$$

b) $$J_1[q_1,q_2,q_3] = \int\limits_{t_1}^{t_2} \left(\dot{q}_1\dot{q}_2 + \dot{q}_2 q_3 + \dot{q}_3 q_1\right) dt$$

### Problem 3.2 (5 Points)

Consider the functional

$$J[y,z] = \int\limits_{x_0}^{x_1} \left(y^2 + z^2\right) dx$$

subject to the constraint

$$y' = z - y.$$

What type of the constaint do we have? Write down the form of the problem including a Lagrange multiplier in the integral. Determine the Euler-Lagrange equations for $y$ and $z$. Solve the equations to find the form of the extremal curve of $J$ under the constraint.

### Problem 3.3 (5 Points)

The Beltrami identity states that the extremal function of the integral

$$I[u] = \int\limits_a^b L(x,u,u')dx$$

satisfies the differential equation

$$\frac{d}{dx}\left(L - u'\frac{\partial L}{\partial u'}\right) - \frac{\partial L}{\partial x} = 0.$$

Please prove the identity using the Euler-Lagrange equation and the chain rule. Note that as a special case, when $L$ does not depend on $x$, we get the equation for the autonomous case, i.e., $H = const.$

**Problem 3.4 (3+3=6 Points)**

Find the extremals of the functionals below subject to the fixed end point conditions prescribed

$$\text{a)} \quad J_1[y] = \int_0^{\pi/2} \left(y^2 + y'^2 - 2y\sin x\right) dx, \quad y(0) = 0, \quad y(\pi/2) = 3/2$$

$$\text{b)} \quad J_2[y] = \int_1^2 \frac{y'^2}{x^3} dx, \quad y(1) = 0, \quad y(2) = 15$$

**Deadline for submission:** Wednesday, June 04, 12 pm