Universität des Saarlandes
Fachrichtung 6.1 – Mathematik
PD Dr. Darya Apushkinskaya

Calculus of Variations (Summer Term 2014)
Assignment H4 - Homework

Problem 4.1 (8 Points)

Minimize the functional

$$J[y] = \int_0^2 (y'+1)^2 (y')^2 dx$$

subject to the end-point conditions that $x(0) = 1$ and $x(2) = 0$.

(*Hint:* consider the possibilty of broken extremals).

Problem 4.2 (9 Points)

Newton's aerodynamic problem (the problem of finding the surface of revolution that minimizes drag) is often approximated by assuming the shape is long and thin, so that y' is large (and negative). In this case approximate

$$\frac{1}{1+(y')^2} \simeq \frac{1}{(y')^2}$$

and the functional of interest by

$$J[y] \simeq \int_0^R \frac{x}{(y')^2} dx,$$

Derive the shape that arise from minimizing this functional.

Problem 4.3 (3x3=9 Points)

Decide which of the following functionals admit broken extremals or not:

$$J_1[y] = \int_0^1 \left(y'^6 - 5y'^4 + 15y'^2\right) dx, \qquad J_2[y] = \int_0^1 y'^3 dx,$$

$$J_3[y] = \int_0^1 \cos\{(y')\}\, dx.$$

(Either construct examples of broken extremals or rule out their existence unsing available theory). Moreover, determine at least one admissible extremal for the minimization of each of the functionals over the set of functions y satisfying $y(0) = 0$ and $y(1) = \pi/2$.

Deadline for submission: Wednesday, June 17, 12 pm