Universität des Saarlandes
Fachrichtung 6.1 – Mathematik
PD Dr. Darya Apushkinskaya

Calculus of Variations (Summer Term 2014)
Assignment H6 - Homework

Problem 6.1 (7 Points)

Minimize

$$J[u] = \int_0^1 u^2 dt$$

subject to

$$\dot{x}_1 = u - x_2$$
$$\dot{x}_2 = -u$$

and

$$x_1(0) = 2$$
$$x_1(1) = 1$$
$$x_2(0) = 0$$
$$x_2(1) = 1$$

Problem 6.2 (8 Points)

Find the minimum value of

$$J[u] = x(1) + \int_0^1 \alpha u^2 dt,$$

where $\alpha > 0$, $x(0) = 0$, $x(1)$ free, and

$$\dot{x} = u.$$

How does the answer change if we add the condition that $|u(t)| \leqslant 1$?

(See next page)

Problem 6.3 (10 Points)

Maximize the range of a missile: Take a missile which has a rocket motor that generates constant thrust f for a fixed time interval $[0, t_1]$. We can control the angle of the thrust $\theta(t)$ (relative to the horizontal). Ignoring drag, the curve of the Earth's surface (and its rotation), determine the angle profile that will maximize the range of the missile.

Hints: choose a coordinates (x, y), and $(u, v) = (\dot{x}, \dot{y})$, then the DEs decribing the system under thrust will be

$$
\begin{aligned}
\dot{x} &= u \\
\dot{y} &= v \\
\dot{u} &= f \cos\theta \\
\dot{v} &= f \sin\theta - g
\end{aligned}
$$

After the rocket stops firing, the missile will continue on a ballistic trajectory, i.e., the remaining motion will be a parabola, resulting in a total firing distance of

$$R(x, y, u, v) = x + \frac{u}{g}\left[v + \sqrt{v^2 + 2gy}\right]$$

where x, y, u, v are given at the time at which ballistic motion commences.

Deadline for submission: Wednesday, July 16, 12 pm