SEMINAR ON COMPLETELY POSITIVE MAPS

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Mandatory reading

The main reference for the seminar is [Pau02]. The book is available in the library and online within the university network (link). Everyone is expected to read Chapter 1 of [Pau02] (Introduction) before the beginning of the seminar.

Talk topics

1. **Positive maps.** Basics on positive maps, von Neumann's inequality: 2.1 – 2.9 + Exercise 2.3 in [Pau02]. Optional: Theorem 2.14 in [Pau02].

2. **Completely positive maps, Part 1.** Unital maps, basics on completely positive maps: 2.10 – 2.13 and 3.1 – 3.6 in [Pau02]. Optional: Theorem 2.7 in [Pau02].

3. **Completely positive maps, Part 2.** Positive maps on commutative C^* algebras, Choi's theorem on completely positive maps between matrix algebras, operators with numerical radius at most one: 3.7 – 3.17 in [Pau02]. Optional: Theorem 3.18 in [Pau02].

4. **Stinespring's dilation theorem.** Stinespring's dilation theorem, Sz.-Nagy's dilation theorem, Choi's dilation theorem: 4.1 – 4.3 and 4.7 in [Pau02]. Optional: Remainder of Chapter 4 in [Pau02].

5. **Completely positive maps into M_n.** Extensions of completely positive maps into M_n, automatic complete positivity; Paulsen 6.1-6.7 and Exercise 2.10. Optional: Remainder of Chapter 6 of [Pau02].

6. **Arveson's extension theorem.** The BW topology, Arveson's extension theorem, applications to spectral sets; 7.1 – 7.8 in [Pau02]. Optional: Remainder of Chapter 7 of [Pau02].

7. **Completely bounded maps.** The off-diagonal technique, Wittstock's extension theorem, Wittstock's dilation theorem: 8.1 – 8.5 in [Pau02]. Optional: "Operator valued measures" in Chapter 8 in [Pau02].

8. **Applications of completely bounded maps.** Bimodule maps, Schur producs: 8.6 – 8.11 in [Pau02]. Optional: Exercises 8.7 and 8.8 in [Pau02].

9. **Commuting contractions.** Dilations of commuting isometries, Ando's theorem, counterexamples in three variables: 5.1 – 5.3, 5.4 – 5.7 in [Pau02]. Optional: Remainder of Chapter 5 in [Pau02].

10. **Completely bounded homomorphisms.** Paulsen's similarity theorem, similarity to *-homomorphisms, Sz-Nagy's theorem on similarity to unitary operators: 9.1 – 9.7 in [Pau02]. Optional: 9.8 – 9.10 in [Pau02].

11. **Similarity to contractions and power bounded operators.** Similiarity to contractions, Rota's theorem, Foguel's example: 9.11 – 9.14 and 10.7 – 10.9 in [Pau02]; see also discussion at the beginning of Chapter 10. Optional: Theorem 10.1 and Exercise 10.1 in [Pau02].

12. **Pisier's counterexample.** Pisier's example of a polynomially bounded operator not similar to a contraction: 10.2 – 10.6 in [Pau02].

13. **Abstract characterization of operator systems.** The Choi–Effros characterization of operator systems: 13.1 – 13.3 in [Pau02]. Optional: 13.4 in [Pau02].

14. **Suggestions by you.** You can suggest a topic for your talk, as long as it fits with the theme of the seminar. In that case, please talk to one of the organizers.

General remarks about your talk

(1) Please prepare a talk of 80 minutes in order to allow 10 minutes of questions.
(2) Please prepare a handout (approximately one page) summarizing the main points of your talk.
(3) Set up a meeting with your contact person at least three weeks before your talk to discuss the details.
(4) Pandemic situation permitting, the seminar will take place in person. You can write on the board (recommended) or use slides.
(5) You may have material for more than 80 minutes. Make a good selection.
(6) It is highly recommended to practice your talk without an audience.

References

[Pau02] Vern Paulsen, *Completely bounded maps and operator algebras*, Cambridge Studies in Advanced Mathematics, vol. 78, Cambridge University Press, Cambridge, 2002. MR 1976867 (2004c:46118)