$\begin{array}{c} & \mbox{Introduction} \\ \mbox{Complement components of amoebas for \mathcal{P}_{Δ}^{*}} \\ \mbox{Geometrical and topological structure of (U_{Δ}^{A})} \\ & \mbox{The Jackpot question for \mathcal{P}_{Δ}^{*}} \end{array}$

The Configuration Space of Amoebas with barycentric Simplex Newton Polytope

Timo de Wolff joint with T. Theobald



5th of July 2011

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Complement components of amoebas for \mathcal{P}^*_{Δ} Geometrical and topological structure of $(U_0^A)^{\mathbb{C}}$ The Jackpot question for \mathcal{P}^*_{Δ}

Introduction

T. de Wolff The Configuration Space of Amoebas with barycentric Simplex N

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 $\begin{array}{l} \text{Complement components of amoebas for \mathcal{P}^*_{Δ}}\\ \text{Geometrical and topological structure of $(U_0^A)^2$}\\ \text{The Jackpot question for \mathcal{P}^*_{Δ}} \end{array}$

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Complement components of amoebas for \mathcal{P}^*_{Δ} Geometrical and topological structure of $(U_0^A)^{\mathbb{C}}$ The Jackpot question for \mathcal{P}^*_{Δ}

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• We investigate Laurent polynomials in $\mathbb{C} [\mathbf{z}^{\pm 1}]$ with $\mathbf{z} := z_1 \cdots z_n$.

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- For fixed exponents we denote the corresponding set as

$$\mathsf{A} := \left\{ \alpha(j) \in \mathbb{Z}^n : 1 \le j \le d \right\}.$$

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$$A:=\left\{\alpha(j)\in\mathbb{Z}^n:1\leq j\leq d\right\}.$$

The NEWTON POLYTOPE New(f) of a Laurent polynomial f is the polytope obtained by taking the convex hull of all its exponent vectors, i.e. here we have New(f) = conv A.

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What is an amoeba?

Complement components of amoebas for \mathcal{P}^*_{Δ} Geometrical and topological structure of $(U_0^{A})^c$ The Jackpot question for \mathcal{P}^*_{Δ}

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Definition (Gelfand, Kapranov, Zelevinsky)

Let $f \in \mathbb{C}[\mathbf{z}]$ with variety $\mathcal{V}(f) \subset (\mathbb{C}^*)^n$.

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Let $f \in \mathbb{C}[\mathbf{z}]$ with variety $\mathcal{V}(f) \subset (\mathbb{C}^*)^n$. Define the Log-map as:

$$\begin{array}{ll} \mathsf{Log}: & (\mathbb{C}^*)^n \to \mathbb{R}^n, \\ & (|z_1| \cdot e^{i \cdot \phi_1}, \dots, |z_n| \cdot e^{i \cdot \phi_n}) \mapsto (\log |z_1|, \dots, \log |z_n|) \end{array}$$

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The AMOEBA $\mathcal{A}(f)$ of f is the image of $\mathcal{V}(f)$ under the Log-map.

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$$f := z_1 + z_2 + 1$$

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• $f := z_1^3 z_2^3 - 9z_1^2 z_2^3 + z_1 z_2^5 - 4z_1 z_2^4 - 4z_1 z_2 + 1$



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Basic properties of amoebas

Let $f \in \mathbb{C}[\mathbf{z}]$ with amoeba $\mathcal{A}(f)$. Then:

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Let $f \in \mathbb{C}[\mathbf{z}]$ with amoeba $\mathcal{A}(f)$. Then:

 A(f) is a closed set with non-empty complement (Gelfand, Kapranov, Zelevinksy).

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- Every configuration of existing and non-existing inner complement components is possible (Rullgård).

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The configuration space of amoebas

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The configuration space of amoebas

• For $A \subset \mathbb{Z}^n$ the CONFIGURATION SPACE \mathbb{C}^A_\diamond is the set

$$\left\{ f \in \mathbb{C}\left[\mathbf{z}^{\pm 1}\right] : f = \sum_{\alpha(j) \in A} b_j \cdot \mathbf{z}^{\alpha(j)}, b_j \in \mathbb{C}, \operatorname{New}(f) = \operatorname{conv} A \right\}$$

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Since every f in C^A_◊ is determined by its coefficient vector, we may interpret C^A_◊ as projective space with dimension d = #A.

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The configuration space of amoebas

Example

Let $f := b_0 + \sum_{j=1}^n b_j \cdot z_j$. Then

$$\mathbb{C}^{\mathcal{A}}_{\diamond} = U^{\mathcal{A}}_{\alpha(0)} = \cdots = U^{\mathcal{A}}_{\alpha(n)}$$

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Theorem (Rullgård)

All U^A_{α} are non-empty, pseudo-convex, semialgebraic sets. The intersection of $(U^A_{\alpha})^c$ with an arbitrary projective line in \mathbb{C}^A_{\diamond} is non-empty and connected.

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Complement components of amoebas for \mathcal{P}^{*}_{Δ} Geometrical and topological structure of $(U^{A}_{0})^{c}$ The Jackpot question for \mathcal{P}^{*}_{Δ}

Key problems on the configuration spaces of amoebas

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• $U^{A}_{\alpha(0)}$ locally has the structure of an area bounded by a hypocycloid.

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- $U^A_{\alpha(0)}$ locally has the structure of an area bounded by a hypocycloid.
- Membership of points in $U_{\alpha(i)}^A$ can be decided for every j.
- All $U^{A}_{\alpha(j)}$ are pathconnected.

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Complement components of amoebas for \mathcal{P}^*_Δ

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 $\begin{array}{l} \text{Complement components of amoebas for } \mathcal{P}_{\Delta}^{\star}\\ \text{Geometrical and topological structure of } (U_{0}^{\star})^{c}\\ \text{The Jackpot question for } \mathcal{P}_{\Delta}^{\star} \end{array}$

Initial Example

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Initial Example

Example (Passare, Rullgård)

Let
$$f := 1 + b_0 \cdot z_1 \cdots z_n + \sum_{j=1}^n z_j^{n+1}$$

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Initial Example

Example (Passare, Rullgård)

Let $f := 1 + b_0 \cdot z_1 \cdots z_n + \sum_{j=1}^n z_j^{n+1}$. Then $\mathcal{A}(f)$ is solid if and only if $\mathcal{A}(f)$ contains the origin which is the case if and only if

$$b_0 \notin K_n := \{-t_0 - \cdots - t_n : t_j \in \mathbb{C}, |t_j| = 1, t_0 \cdots t_n = 1\}.$$

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 K_2 looks approximately the following way:



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The class of investigated polynomials

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The class of investigated polynomials

Definition

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The class of investigated polynomials

Definition

Let P_∆ be the class f := b₀ + ∑_{j=1}ⁿ⁺¹ b_i ⋅ z^{α(j)} of Laurent-polynomials with conv{α(1),...,α(n + 1)} being a simplex containing the origin in the interior.

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- 2 Let $\mathcal{P}^*_{\Delta} \subset \mathcal{P}_{\Delta}$ be the subclass with $\alpha(1) = -\sum_{j=2}^{n+1} \alpha(j)$.

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The equilibrium point

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The equilibrium point

Definition

Let $f \in \mathcal{P}_{\Delta}$. We define the EQUILIBRIUM POINT $eq(f) \subset \mathbb{R}^n$ as the point with

$$|b_1 \cdot (\mathrm{Log}^{-1}(\mathrm{eq}(f)))^{\alpha(1)}| = \cdots = |b_{n+1} \cdot (\mathrm{Log}^{-1}(\mathrm{eq}(f)))^{\alpha(n+1)}|.$$

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Remark

eq(f) is the unique vertex of the tropical variety $T(Trop(f - b_0))$ obtained by "naive" tropicalization via Log-valuation

$$\mathsf{Trop}(f - b_0) := \bigoplus_{j=1}^{n+1} \log |b_j| \odot \langle \mathsf{Log}(\mathbf{z}), \alpha(j) \rangle$$

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Initial example II

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Initial example II

Example

Let
$$f_{b_0} := 1 + b_0 \cdot z_1 \cdots z_n + \sum_{j=1}^n z_j^{n+1}$$
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Initial example II

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Let
$$f_{b_0} := 1 + b_0 \cdot z_1 \cdots z_n + \sum_{j=1}^n z_j^{n+1}$$
. After rescaling by $f \to f \cdot \mathbf{z}^{-(1,\dots,1)}$ we have $f \in \mathcal{P}^*_{\Delta}$.

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 $\left| \log^{-1}(\mathbf{0})^{-(1,\dots,1)} \right| = \left| \log^{-1}(\mathbf{0})^{(0,\dots,0,n+1,0,\dots,0)} \right| = 1.$

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The inner complement component of $\mathcal{A}(f)$ with $f \in \mathcal{P}^*_{\Delta}$

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The inner complement component of $\mathcal{A}(f)$ with $f \in \mathcal{P}^*_{\Delta}$

Theorem (Theobald, dW.)

Let f_{b_0} be a family of parametric polynomials in \mathcal{P}^*_{Δ} with parameter $b_0 \in \mathbb{C}$.

The inner complement component of $\mathcal{A}(f)$ with $f \in \mathcal{P}^*_{\Delta}$

Theorem (Theobald, dW.)

Let f_{b_0} be a family of parametric polynomials in \mathcal{P}^*_{Δ} with parameter $b_0 \in \mathbb{C}$. Then the following statements are equivalent:

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The inner complement component of $\mathcal{A}(f)$ with $f \in \mathcal{P}^*_{\Delta}$

Theorem (Theobald, dW.)

Let f_{b_0} be a family of parametric polynomials in \mathcal{P}^*_{Δ} with parameter $b_0 \in \mathbb{C}$. Then the following statements are equivalent: $f_{b_0} \in U^A_0$ (i.e. $\mathcal{A}(f_{b_0})$ has genus 1),

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The inner complement component of $\mathcal{A}(f)$ with $f \in \mathcal{P}^*_{\Delta}$

Sketch of proof for $(1) \Rightarrow (2)$:

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The inner complement component of $\mathcal{A}(f)$ with $f \in \mathcal{P}^*_\Delta$

Sketch of proof for $(1) \Rightarrow (2)$:

• For every $\mathbf{w} \in \mathbb{R}^n$ there is a fiberfunction

$$\mathcal{F}_{\mathbf{w},f_{b_0}}:[0,2\pi)^n \to \mathbb{C}, \phi \mapsto f(\mathrm{Log}^{-1}(\mathbf{w}),\phi).$$

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• Let $\mathbf{w} \in E_{\mathbf{0}}(f_{b_0})$ with $\mathbf{w} \neq eq(f)$.

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• Let
$$\mathbf{w} \in E_0(f_{b_0})$$
 with $\mathbf{w} \neq eq(f)$.

• Investigate the tropical hypersurface \mathcal{T} given by $\bigoplus_{j=1}^{n+1} \log |b_j| \odot \langle \text{Log}(\mathbf{z}), \alpha(j) \rangle$

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• Let
$$\mathbf{w} \in E_{\mathbf{0}}(f_{b_0})$$
 with $\mathbf{w} \neq eq(f)$.

Let A_0, \ldots, A_n be the regions of $\mathbb{R}^n \setminus T$ and let w.l.o.g $\mathbf{w} \in A_0$.

Key step: For every A_j ≠ A₀ there exists a w(j) ∈ ℝⁿ such that there is an isomorphism π_{w(j)} on [0, 2π)ⁿ with

$$\mathcal{F}_{\mathbf{w},f_{b_0}}(\phi) = \mathcal{F}_{\mathbf{w}(j),f_{b_0}}(\pi_{\mathbf{w}(j)}(\phi)).$$

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• Thus, in particular $\mathbf{w} \in E_{\mathbf{0}}(f_{b_0}) \Rightarrow \mathbf{w}(j) \in E_{\mathbf{0}}(f_{b_0})$ for all j.

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- Thus, in particular $\mathbf{w} \in E_{\mathbf{0}}(f_{b_0}) \Rightarrow \mathbf{w}(j) \in E_{\mathbf{0}}(f_{b_0})$ for all j.
- Hence, $\mathbf{w} \in E_0(f_{b_0}) \Rightarrow \operatorname{conv}\{\mathbf{w}, \mathbf{w}(1), \dots, \mathbf{w}(n)\} \subset E_0(f_{b_0})$ since $E_0(f_{b_0})$ is convex.

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The inner complement component of $\mathcal{A}(f)$ with $f \in \mathcal{P}^*_\Delta$

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- Hence, $\mathbf{w} \in E_0(f_{b_0}) \Rightarrow \operatorname{conv}\{\mathbf{w}, \mathbf{w}(1), \dots, \mathbf{w}(n)\} \subset E_0(f_{b_0})$ since $E_0(f_{b_0})$ is convex.
- Therefore, w ∈ E₀(f_{b0}) ⇒ eq(f) ∈ E₀(f_{b0}) since eq(f) is unique vertex of T.

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The inner complement component of $\mathcal{A}(f)$ with $f \in \mathcal{P}^*_{\Lambda}$

Theorem (Theobald, dW.)

Let f_{b_0} be a family of parametric polynomials in \mathcal{P}^*_{Δ} with parameter $b_0 \in \mathbb{C}$. Then the following statements are equivalent:

•
$$f_{b_0} \in U_0^A$$
 (i.e. $\mathcal{A}(f_{b_0})$ has genus 1),

• $eq(f) \in E_0(f_{b_0})$,

• $b_0 \notin \left\{ -|\Theta| \cdot \sum_{j=0}^n e^{j \cdot (\arg(b_j) + \langle \alpha(j), \phi \rangle)} : \phi \in [0, 2\pi)^n \right\}$.

Aim: Describe the geometric struture of the set $S := \left\{ b_0 \in \mathbb{C} : \mathcal{V}(\mathcal{F}_{eq(f), f_{b_0}}) \neq \emptyset \right\}$

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Geometrical and topological structure of $(U_{\mathbf{0}}^{A})^{c}$

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Hypocycloids

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Hypocycloids

Definition

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Hypocycloids

Definition

For R > r, a HYPOCYCLOID with parameters $R, r \in \mathbb{R}_{>0}$ is the parametric curve in $\mathbb{R}^2 \cong \mathbb{C}$ given by

$$(R-r)\cdot e^{i\cdot\phi}+r\cdot e^{i\cdot\left(\frac{r-R}{r}\right)\cdot\phi}, \quad \phi\in[0,2\pi).$$

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Geometrically, it is the trajectory of some fixed point on a circle with radius r rolling (from the interior) on a circle with radius R.

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 $\begin{array}{c} \\ \mbox{formula} & \mbox{formula} \\ \mbox{Geometrical and topological structure of } (U_{0}^{A})^{c} \\ \\ \mbox{The Jackpot question for } \mathcal{P}_{\Delta}^{*} \end{array}$

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Geometrical and topological structure of $(U_0^A)^c$

Theorem (Theobald, dW.)

Let $A := \{\mathbf{0}, \alpha(1), \dots, \alpha(n+1)\} \subset \mathbb{Z}^n$ such that conv A is a simplex with barycenter $\mathbf{0}$.

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Geometrical and topological structure of $(U_0^A)^c$

Theorem (Theobald, dW.)

Let $A := \{\mathbf{0}, \alpha(1), \dots, \alpha(n+1)\} \subset \mathbb{Z}^n$ such that conv A is a simplex with barycenter $\mathbf{0}$. For $b_1, \dots, b_{n+1} \in \mathbb{C}^*$ the intersection of $\partial (U_{\mathbf{0}}^A)^c$ with $h := \{(b_0 : b_1 : \dots : b_{n+1}) : b_0 \in \mathbb{C}\}$ is the hypocycloid with parameters $R = (n+1) \cdot |\Theta|, r = |\Theta|$ and cusps at

$$\arg(b_0) = \pi \cdot \left(1 + \frac{2k - \sum_{i=1}^n \arg(b_i)}{n+1}\right), \quad k \in \{0, \dots, n\}.$$

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Geometrical and topological structure of $(U_0^A)^c$

Theorem (Theobald, dW.)

Let $A := \{\mathbf{0}, \alpha(1), \dots, \alpha(n+1)\} \subset \mathbb{Z}^n$ such that conv A is a simplex with barycenter $\mathbf{0}$. For $b_1, \dots, b_{n+1} \in \mathbb{C}^*$ the intersection of $\partial (U_{\mathbf{0}}^A)^c$ with $h := \{(b_0 : b_1 : \dots : b_{n+1}) : b_0 \in \mathbb{C}\}$ is the hypocycloid with parameters $R = (n+1) \cdot |\Theta|, r = |\Theta|$ and cusps at

$$\arg(b_0) = \pi \cdot \left(1 + \frac{2k - \sum_{i=1}^n \arg(b_i)}{n+1}\right), \quad k \in \{0, \dots, n\}.$$

Furthermore, $(U_0^A)^c$ is simply connected.

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Example

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Example

Example

We investigate the parametric family of polynomials

$$f_{b_0} = b_0 + z_1^{-1} z_2^{-3} + 2.4 \cdot z_1^1 z_2^{-2} + (1 + 1.3i) \cdot z_2^5.$$

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Example

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$$f_{b_0} = b_0 + z_1^{-1} z_2^{-3} + 2.4 \cdot z_1^1 z_2^{-2} + (1 + 1.3i) \cdot z_2^5.$$

Then $\mathbb{C}^{\mathcal{A}}_{\diamond} \cap \{(b_0 : 1 : 2.4 : 1 + 1.3 \cdot i) : b_0 \in \mathbb{C}\}$ looks approximately the following way:



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Sketch of proof

• By former theorem: $\partial S = \partial (U^A_{\alpha(0)})^c \cap h$ with

$$S = \left\{ b_0 \in \mathbb{C} : \mathcal{V}(\mathcal{F}_{eq(0), f_{b_0}}) \neq \emptyset \right\}.$$

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Sketch of proof

• By former theorem:
$$\partial S = \partial (U^A_{\alpha(0)})^c \cap h$$
 with
 $S = \left\{ b_0 \in \mathbb{C} : \mathcal{V}(\mathcal{F}_{eq(0), f_{b_0}}) \neq \emptyset \right\}.$

• Let
$$k := -n + 1 + (-1)^{n+1}$$
 and
 $F : [k, n] \times [0, 2\pi) \rightarrow \mathbb{C},$
 $(\mu, \psi) \mapsto |\Theta| \cdot \mu \cdot e^{i \cdot \psi} + |\Theta| \cdot e^{i \cdot (-n \cdot \psi + \sum_{j=1}^{n} \arg(b_j))}.$

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Complement components of amoebas for \mathcal{P}^*_{Λ} Geometrical and topological structure of $(U_0^{\Lambda})^{c}$ The Jackpot question for \mathcal{P}^*_{Λ}

Sketch of proof

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1 The image of F is contained in S,

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Sketch of proof

• By former theorem:
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Sketch of proof

 $\partial T \subseteq \partial S$.

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• By former theorem:
$$\partial S = \partial (U^A_{\alpha(0)})^c \cap h$$
 with
 $S = \left\{ b_0 \in \mathbb{C} : \mathcal{V}(\mathcal{F}_{eq(0), f_{b_0}}) \neq \emptyset \right\}.$

• Let
$$k := -n + 1 + (-1)^{n+1}$$
 and
 $F : [k, n] \times [0, 2\pi) \rightarrow \mathbb{C},$
 $(\mu, \psi) \mapsto |\Theta| \cdot \mu \cdot e^{i \cdot \psi} + |\Theta| \cdot e^{i \cdot (-n \cdot \psi + \sum_{j=1}^{n} \operatorname{arg}(b_j))}.$

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Sketch of proof

• Let T denote the region whose boundary is the hypocycloid given by $\phi \mapsto F(n, \phi)$ for $\phi \in [0, 2\pi)$. Then $S \subseteq T$ and $\partial T \subseteq \partial S$.

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Sketch of proof

- Let T denote the region whose boundary is the hypocycloid given by $\phi \mapsto F(n, \phi)$ for $\phi \in [0, 2\pi)$. Then $S \subseteq T$ and $\partial T \subseteq \partial S$.
- The set S equals the region T whose boundary is the hypocycloid with parameter R = (n + 1) · |Θ|, r = |Θ| given by φ → F(n, φ) for φ ∈ [0, 2π). In particular, S is simply connected.

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Sketch of proof

- Let T denote the region whose boundary is the hypocycloid given by $\phi \mapsto F(n, \phi)$ for $\phi \in [0, 2\pi)$. Then $S \subseteq T$ and $\partial T \subseteq \partial S$.
- The set *S* equals the region *T* whose boundary is the hypocycloid with parameter $R = (n+1) \cdot |\Theta|$, $r = |\Theta|$ given by $\phi \mapsto F(n, \phi)$ for $\phi \in [0, 2\pi)$. In particular, *S* is simply connected.



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The Jackpot question for \mathcal{P}^*_Δ

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The Jackpot question for \mathcal{P}^*_Δ

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The Jackpot question for \mathcal{P}^*_Δ

Theorem (Theobald, dW.)

Let $\mathbb{C}^A \subset \mathcal{P}^*_{\Delta}$. Then U^A_0 is pathconnected.

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The Jackpot question for \mathcal{P}^*_Δ

Theorem (Theobald, dW.)

Let $\mathbb{C}^A \subset \mathcal{P}^*_{\Delta}$. Then U^A_0 is pathconnected.

It suffices to show:

Theorem (Theobald, dW.)

Let $A := \{\alpha(1), \ldots, \alpha(d)\}$. If for every *b* the set

$$\{(\lambda \cdot b_1 : \cdots : b_d) : \lambda \in \mathbb{C}\} \cap (U^A_{\alpha(1)})^{\alpha}$$

is simply connected, then $U^{\mathcal{A}}_{\alpha(1)}$ is pathconnected.

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Proof

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Proof

• Let
$$b, b' \in U^{\mathcal{A}}_{\alpha(1)} \subset \mathbb{C}^{\mathcal{A}}_{\diamond}$$
 in complex lines g, g' .



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Proof

•
$$g \cap \left(U_{\alpha(1)}^{A}\right)^{c}$$
, $g' \cap \left(U_{\alpha(1)}^{A}\right)^{c}$ simply connected.



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Proof

• Investigate line segment $\sigma(b, b') := b - \lambda \cdot (b' - b), \lambda \in [0, 1].$



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Proof

∀p ∈ σ(b, b') ⊂ C^A_◊ compute minimal value of |b₁|, such that ∃ w ∈ ℝⁿ with f_p{w} being lopsided with dominating term b₁ ⋅ z^{α(1)}.



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Proof

• Take the maximum $|\tilde{b_1}|$ of all these values.



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Proof

• Take the maximum $|\tilde{b_1}|$ of all these values.



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Proof

• Investigate the points \tilde{b}, \tilde{b}' ;



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Proof

• Investigate the points \tilde{b}, \tilde{b}' ; construct pathes γ_1 from b to \tilde{b} and γ_2 from \tilde{b}' to b'.



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Proof

• γ_1 can be constructed in $U^A_{\alpha(1)}$ since $b, \tilde{b} \in U^A_{\alpha(1)}$ and $(U^A_{\alpha(1)})^c \cap g$ is simply connected (analogue for γ_2).



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Proof

• Investigate \tilde{b}'' ;

$$Im(b_1) \begin{array}{c} b_2, \dots, b_d \end{array} \qquad \tilde{b}'' := (|\tilde{b}_1'| \cdot \arg(b_1') + 1 : |b_2| \cdot \arg(b_2') : \dots : |b_d| \cdot \arg(b_d'))$$

Re(b_1)

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Proof

• Investigate \tilde{b}'' ; construct path γ_3 from \tilde{b} to \tilde{b}'' in $T_{\tilde{b}} := \{(e^{i \cdot \phi_1} \cdot |\tilde{b_1}| \cdot \arg(b_1) : e^{i \cdot \phi_2} \cdot b_2 : \cdots : e^{i \cdot \phi_d} \cdot b_d) : \phi_1, \ldots, \phi_d \in [0, 2\pi)\}.$



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Proof

• $\gamma_3 \in U^A_{\alpha(1)}$ since: if $f_b\{\mathbf{w}\}$ is lopsided for some $\mathbf{w} \in \mathbb{R}^n$, then $f_{b'}\{\mathbf{w}\}$ is lopsided for every $b' \in T_{f_b}$.



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Proof

• Construct γ_4 as line segment $\tilde{b}'' + \lambda \cdot (\tilde{b}' - \tilde{b}'')$, $\lambda \in [0, 1]$.



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Proof

• γ_4 lies on affine line $\tilde{b}'' + \mu \cdot (0, |b_2| - |b_2|', \dots, |b_d| - |b_d|')$. Every point of this line between \tilde{b}'' and \tilde{b}' is lopsided due to construction of $|\tilde{b_1}|$. Hence, $\gamma_4 \in U^A_{\alpha(1)}$.



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Proof

• Thus, $\gamma := \gamma_2 \circ \gamma_4 \circ \gamma_3 \circ \gamma_1$ is contained in $U^A_{\alpha(1)}$ and therefore *b* and *b'* are pathconnected.



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Thank you for your attention!

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