Tensors in Computer Science
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Sheet No. 01
To be discussed on Thursday, 7 May

1. Show that $\mathbb{Z}/a \otimes \mathbb{Z}/b \cong \mathbb{Z}/\gcd(a,b)$ for every $a, b \in \mathbb{Z}$.

2. Let $A \in \mathbb{Z}^{n \times n}$ be an integer matrix with $\det(A) \neq 0$ and let $G = \operatorname{coker}(\mathbb{Z}^n \xrightarrow{A} \mathbb{Z}^n)$. Prove that $|G| = |\det(A)|$.

3. Let R be any ring. Define
$$R[[x]] = \{\sum_{n=0}^{\infty} a_n x^n : a_n \in R\},$$
the ring of general power series with coefficients in R, with sum and product defined as
$$(\sum_{n=0}^{\infty} a_n x^n) + (\sum_{n=0}^{\infty} b_n x^n) = \sum_{n=0}^{\infty} (a_n + b_n) x^n,$$
$$(\sum_{n=0}^{\infty} a_n x^n) \cdot (\sum_{n=0}^{\infty} b_n x^n) = \sum_{n=0}^{\infty} \sum_{p+q=n} (a_p b_q) x^n.$$
Prove that R Noetherian implies $R[[x]]$ Noetherian.

4. Prove that $\mathcal{C}[0,1]$, the ring of continous functions defined on $[0,1]$, is not Noetherian.