



Algebraic Geometry
Summer Term 2018

Exercise Sheet 4. Hand in by Friday, May 18.

Exercise 1

Let $M \subset K[x_1, \dots, x_n]$ be a finite set of monomials, and let $>$ be a global monomial order. Prove that there exists a weight vector $w = (w_1, \dots, w_n) \in \mathbb{R}_{>0}^n$ such that the partial weight order $>_w$ induces the same order on M than the given order $>$. Show that one can take w even with integer coefficients.

Exercise 2

Prove Dixon's Lemma by induction on n without the use of Hilbert's basis theorem.

Exercise 3

Using Buchberger Criterion, prove that the 2×2 minors of

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{pmatrix}$$

form a Gröbner basis with respect to $>_{lex}$ and

$$x_1 > \dots > x_4 > y_1 > \dots > y_4.$$

Exercise 4

Install the Computer algebra package Macaulay2
<https://faculty.math.illinois.edu/Macaulay2/>
on your laptop.