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Algebraic Geometry Summer Term 2018

Exercise Sheet 7. Hand in by Friday, June 8.

Exercise 1 (Localization commutes with passing to quotients) Let $U \subset R$ be a multiplicative subset of a ring, and let $I \subset R$ be an ideal. Consider the image $\overline{U}$ of U under the quotient map $R \to R/I$. Then

$$R/I[\overline{U}^{-1}] \cong R[U^{-1}]/IR[U^{-1}].$$

Exercise 2
Let $B_1, B_2 \subset \mathbb{A}^n$ be algebraic sets, $A = B_1 \cup B_2$ and let $p \in A \setminus B_2$. Prove $\mathcal{O}_{A,p} \cong \mathcal{O}_{B_1,p}$.

Exercise 3

(1) Let $U \subset R$ be a multiplicative subset, and let M be an R-module. Define $M[U^{-1}]$ and prove that this is an $R[U^{-1}]$-module.

(2) Let
$$0 \to M' \to M \to M'' \to 0$$
be a short exact sequence of R-modules. Prove that the induced sequence
$$0 \to M'[U^{-1}] \to M[U^{-1}] \to M''[U^{-1}] \to 0$$
is short exact as well.

Exercise 4
Let $P = K[x_1, x_2]$ be a polynomial ring in two variables.

(1) Find an ideal $I \subset P$ and two minimal generating sets of I with a different number of elements.

(2) Let $n \in \mathbb{N}$ be integer. Find an ideal $J_n \subset P$ which cannot be generated by fewer than n elements.