UNIVERSITÄT DES SAARLANDES Fachrichtung 6.1 - Mathematik Prof. Dr. Frank-Olaf Schreyer

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Exercises Algebraic Geometry

Winterterm 2016/17

The solutions are collected on Tuesday, before the exercise session. All further informations concerning the lecture can be found here: https://www.math.unisb.de/ag/schrever/index.php/teaching

Sheet 6

Exercise 1 (3.2.26). Show that the following rings are integral domains, and find their normalizations:

- (1) The coordinate ring of the plane curve $V(y^2 x^{2k+1}) \subset \mathbb{A}^2$, where $k \geq 1$.
- (2) The coordinate ring of the Whitney umbrella $V(x^2 y^2 z) \subset \mathbb{A}^3$.

Exercise 2 (3.3.10). Let $A \subset \mathbb{A}^n$ be an algebraic set. Show that A is a hypersurface iff it is equidimensional of dimension n-1.

Exercise 3 (3.4.4). Let $I \subset S = \mathbb{k}[x_1, \ldots, x_4]$ be the ideal which is generated by the 2 × 2 minors of the matrix ,

$$\begin{pmatrix} x_1 & x_2 & x_3 \\ x_2 & x_3 & x_4 \end{pmatrix}$$

Find a Noether normalization as in Theorem 3.4.3.

Exercise 4 (4.1.5). (1) Find all singular points of the curve

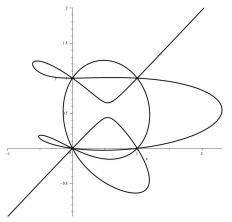
$$V(x^{2} - 2x^{3} + x^{4} + y^{2} - 2y^{3} + y^{4} - \frac{3}{2}x^{2}y^{2}) \subset \mathbb{A}^{2}(\mathbb{C}).$$

Draw a picture of the real points of this curve.

(2) Find all singular points of the curve $V(f) \subset \mathbb{A}^2(\mathbb{C})$, where

$$\begin{split} f &= 11\,y^7 + 7\,y^6x + 8\,y^5x^2 - 3\,y^4x^3 - 10\,y^3x^4 - 10\,y^2x^5 - x^7 - 33\,y^6 \\ &- 29\,y^5x - 13\,y^4x^2 + 26\,y^3x^3 + 30\,y^2x^4 + 10\,yx^5 + 3\,x^6 + 33\,y^5 \\ &+ 37\,y^4x - 8\,y^3x^2 - 33\,y^2x^3 - 20\,yx^4 - 3\,x^5 - 11\,y^4 - 15\,y^3x \\ &+ 13\,y^2x^2 + 10\,yx^3 + x^4, \end{split}$$

is the degree-7 polynomial considered in Example 1.2.4, part 3.





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