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Mathematics for computer science 1

Winterterm 2019/20

This exercise sheet is conducive to a preparation for the exam and will not be discussed in the exercise sessions.

Sheet 13 29. January 2020

Exercise 1 (Taylor polynomial)**.** Determine (without a computer) the Taylor polynomial of order 3 of $g : \mathbb{R} \to \mathbb{R}$, $x \mapsto g(x) = e^{\sqrt{x}}$ in $x_0 = 1$.

Exercise 2 (Taylor series)**.** Compute the Taylor series of

$$f(x) = \frac{x}{(x-1)(x+1)}$$

in $x_0 = 0$.
Show that the Taylor series converges to f in the open interval $(-1, 1)$ (Hint: use partial fraction decomposition).

Exercise 3 (Taylor polynomials and computer algebra)**.** Compute, e.g. with *Maple*, the Taylor polynomials T_0^k of order $k = 1, ..., 6$ in $x_0 = 0$ for

(a) $f(x) = \tan(x)$
(b) $f(x) = \sqrt{1+x}$

and plot the graphs of f and the Taylor polynomials.

Exercise 4 (Improper integral)**.** Determine all $s \in \mathbb{R}$ for which the following improper integral exists

$$\int_1^\infty \frac{1}{x(\ln x)^s} dx.$$

Exercise 5 (Series)**.** Determine all $x \in \mathbb{R}$ for which the series

$$\sum_{n=1}^\infty n^2 x^n$$

converges and compute in these cases the limit.

Exercise 6 (Limits)**.** Compute the following limits, if existent.

(a) $\lim\limits_{x \to \frac{\pi}{2}} \tan(x)(x - \frac{\pi}{2})$
(b) $\lim\limits_{x \to 0} \ln(x) e^{-x}$
(c) $\lim\limits_{x \to \infty} \ln(x) e^{-x}$
(d) $\lim\limits_{x \to \infty} \frac{\sqrt{e^x}}{e^{\sqrt{x}}}$

Exercise 7 (Arithmetic and geometric average)**.** Let $a_0, b_0 \in \mathbb{R}$ with $0 < a_0 < b_0$. The sequences $(a_i)_{i \in \mathbb{N}}$ and $(b_i)_{i \in \mathbb{N}}$ are defined recursively by

$$a_{i+1} = \sqrt{a_i b_i} \quad \text{and} \quad b_{i+1} = \frac{a_i + b_i}{2}, \text{ respectively.}$$

Show that both sequences are convergent and their limits coincide.

Exercise 8 (Induction).

(a) Show that for all numbers $n \geq 2$ the following formula holds.

$$\prod_{k=2}^{n} \left(1 - \frac{2}{k(k+1)}\right) = \frac{1}{3}\left(1 + \frac{2}{n}\right).$$

(b) Show that for all numbers $n \geq 1$ the following formula holds.

$$\sum_{k=0}^{n-1} (n+k)(n-k) = \frac{n(n+1)(4n-1)}{6}$$

Exercise 9 (Logic).
The three students Amann, Bemann and Cemann have a discussion on the solution of an exercise. All of them have a different Ansatz to solve the exercise. The three students do not know which solutions they should hand in and therefore, they ask advice from their tutor. Without solving the exercise for the students, the tutor gives the following hints:

- If Cemann solves the exercise correctly, then Bemann's solution is wrong.
- If Bemann solves the exercise correctly or Amann does not, then Cemann's solution is correct.
- If Bemann or Amann solve the exercise correctly, then Cemann does not solve the exercise correctly.

Decide which solution should be handed in.

Exercise 10 (Extrema). Squares of the same size are cutted out at the corners from a rectangular piece of sheet metal with a side length of 16cm and height of 10cm (see graphic below). The rest of the sheet is bent into a box. How long must the side length of the squares be, such that the volume of the box will be maximal.