Fakultät MI, Fachrichtung Mathematik
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UNIVERSITÄT DES SAARLANDES

Mathematics for computer science 1

Winterterm 2019/20

Hand in your solution sheet in the mailboxes (next to Zeichensaal U.39, building E2 5) by Nov. 06 **before the lecture**.

All exercise sheets and course information can be found at: www.math.uni-sb.de/ag/schreyer/

Sheet 2 30. October 2019

Exercise 1 (Pigeonhole principle and cartesian product)**.** Let $a_1, \dots, a_{101} \in \mathbb{Z}$ be 101 pairwise distinct numbers. Show that there exists a subsequence $a_{i_1}, a_{i_2}, \dots, a_{i_{11}}$ for $i_1 < \dots < i_{11}$ of 11 numbers such that the subsequence is either monotonically decreasing ($a_{i_1} > a_{i_2} > \dots > a_{i_{11}}$) or monotonically increasing ($a_{i_1} < a_{i_2} < \dots < a_{i_{11}}$).

Exercise 2 (Injectivity and surjectivity)**.** Let M and N be finite sets. How many injective maps from M to N exist? How many surjective maps from M to N exist, if N contains two, three or four elements? Do you have an idea for the general case $|N| = n \in \mathbb{N}$.

Exercise 3 (Equivalence relations)**.** We define on $M = \mathbb{N} \times \mathbb{N}$ a relation $\sim$ by

$$(a, b) \sim (c, d) \iff a + d = b + c$$

(a) Show that $\sim$ is an equivalence relation on M.
(b) Describe the equivalence classes $[(1,1)]$ and $[(3,1)]$.
(c) We define an addition on equivalence classes in $M/\sim$ by

$$[(a, b)] + [(c, d)] = [(a + c, b + d)].$$

Show the welldefinedness, that is, show that for $(a, b) \sim (a', b')$ and $(c, d) \sim (c', d')$ also $(a + c, b + d) \sim (a' + c', b' + d')$ holds.
(d) The set $M/\sim$ together with the above defined addition is a well-known set. What is the name of this set? How can you define a multiplication on $M/\sim$?

Exercise 4 (Binomial coefficients)**.** Show that for all $n, k, s, t \in \mathbb{N}$ the following two equations are satisfied.

$$(1) \qquad (k+1)\binom{n}{k+1} = (n-k)\binom{n}{k},$$

$$(2) \qquad \binom{s+t}{n} = \sum_{i=0}^{n} \binom{s}{i}\binom{t}{n-i}.$$

Provide an interpretation of the equations (1) and (2) in terms of the definition of the binomial coefficient.