**Fakultät MI, Fachrichtung Mathematik**
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UNIVERSITÄT DES SAARLANDES

## Mathematics for computer science 1

Winterterm 2019/20

Hand in your solution sheet in the mailboxes (next to Zeichensaal U.39, building E2 5) by Nov. 27 **before the lecture**.

All exercise sheets and course information can be found at: www.math.uni-sb.de/ag/schreyer/

**Sheet 5** 20. November 2019

**Exercise 1.** For $n \in \mathbb{N}$ we define the sequences:

$$\begin{aligned} a_n &= \sqrt{n+1000} - \sqrt{n}, \\ b_n &= \sqrt{n+\sqrt{n}} - \sqrt{n}, \\ c_n &= \sqrt{n+\frac{n}{1000}} - \sqrt{n}. \end{aligned}$$

Show that for $1 \leq n < 1.000.000$ we have $a_n > b_n > c_n$, but

$$\lim_{n\to\infty} a_n = 0, \text{ and } \lim_{n\to\infty} b_n = \frac{1}{2}$$

and the sequence $(c_n)_{n\in\mathbb{N}}$ is not bounded.

**Exercise 2** (Landau–symbols)**.** Which of the following statements are true?

(a) $\frac{n^3+n+1}{2n^2-5} \in O(n)$.
(b) $\frac{n^3+n+1}{2n^2-5} \in o(n)$.
(c) $\frac{2n-5}{20n\sqrt{n}+1000} \in O\left(\frac{1}{n}\right)$.
(d) $\frac{20n\sqrt{n}+1000}{2n-5} \in O(n)$.

**Exercise 3** (Convergence of a sequence)**.** We define the sequence $(a_n)_{n\in\mathbb{N}_0}$ by $a_0 = 1$ and

$$a_n = \sqrt{1+a_{n-1}} \quad \forall n \geq 1.$$

Show that the sequence converges and compute its limit. (Hint: Compute the hypothetical limit.)

**Exercise 4.** (a) Let $(a_n)$ be the Fibonacci-sequence defined by $a_{n+2} = a_{n+1} + a_n$ where $a_1 = 1$ and $a_0 = 0$.

1. Show that $f_n = \frac{1}{\sqrt{5}}\big((\frac{1+\sqrt{5}}{2})^n - (\frac{1-\sqrt{5}}{2})^n\big)$ satisfies the above recursion.
2. Compute the limit $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n}$.

(b) Let $(a_n)$ be the sequence defined by $a_{n+2} = 2a_{n+1} + a_n$ where $a_1 = 1$ and $a_0 = 0$. Compute the limit $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n}$.