Fakultät MI, Fachrichtung Mathematik
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UNIVERSITÄT DES SAARLANDES

Mathematics for computer science 1

Winterterm 2019/20

Hand in your solution sheet in the mailboxes (next to Zeichensaal U.39, building E2 5) by Dec. 04 **before the lecture**.

All exercise sheets and course information can be found at: www.math.uni-sb.de/ag/schreyer/

Sheet 6 27. November 2019

Exercise 1. Show that $\mathbb{Q}[\sqrt{2}] := \{a+b\sqrt{2} \mid a, b \in \mathbb{Q}\} \subset \mathbb{R}$ together with the two operations

$$+ : \mathbb{Q}[\sqrt{2}] \times \mathbb{Q}[\sqrt{2}] \longrightarrow \mathbb{Q}[\sqrt{2}],\ (a + b\sqrt{2}, c + d\sqrt{2}) \mapsto (a + c) + (b + d)\sqrt{2},$$
$$\cdot : \mathbb{Q}[\sqrt{2}] \times \mathbb{Q}[\sqrt{2}] \longrightarrow \mathbb{Q}[\sqrt{2}],\ (a + b\sqrt{2}, c + d\sqrt{2}) \mapsto (ac + 2bd) + (ad + bc)\sqrt{2},$$

is a field. (Hint: in order to compute the inverse, use the third binomial formula and expansion of fractions.)

Exercise 2 (Infimum und supremum)**.** Let

$$M_1 = \{x \in \mathbb{Q} \mid x^2 < 2\} \subset \mathbb{R} \text{ and let } M_2 = \{x \in \mathbb{Q} \mid x^2 > 2\} \subset \mathbb{R}.$$

Which of these sets has a supremum, which an infimum? What is the supremum/infimum if existent?

Exercise 3 (Countability)**.** Show that

(a) the set of all finite subsets of $\mathbb{N}$ is countable.
(b) the set of all subsets of $\mathbb{N}$ is uncountable.

Exercise 4 (Limits of series)**.** Determine which of the following series are convergent and compute their limits, if existent.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{4^{n+1}}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2 - 1}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n + 4}{n^2 - 3n + 1}$