Fakultät MI, Fachrichtung Mathematik
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UNIVERSITÄT DES SAARLANDES

Mathematics for computer science 1

Winterterm 2019/20

Hand in your solution sheet in the mailboxes (next to Zeichensaal U.39, building E2 5) by Dec. 11 **before the lecture**.

All exercise sheets and course information can be found at: www.math.uni-sb.de/ag/schreyer/

Sheet 7 04. December 2019

Exercise 1 (Convergence of series)**.** Let (a_n) be a monotonously decreasing sequence in $\mathbb{R}_{>0}$. Show that $\sum_{n=0}^{\infty} a_n$ is convergent if and only if $\sum_{k=0}^{\infty} 2^k a_{2^k}$ is convergent.

Exercise 2 (Rearrangement)**.** Let $\sum_{n=0}^{\infty} a_n$ be a convergent, but not absolutely convergent series. Show that

(a) the subseries of positive/negative entries is an unbounded increasing/decreasing series, respectively.

(b) for all real numbers $a \in \mathbb{R}$ there exists a rearrangement $\tau\colon \mathbb{N}_0 \to \mathbb{N}_0$, such that $\sum_{n=0}^{\infty} a_{\tau(n)}$ has the limit a.

Exercise 3 (Convergence)**.** Determine the radius of convergence of the following series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

(b) $\displaystyle\sum_{n=1}^{\infty} n^2 x^n$

(c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{x}{2}\right)^n$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{3}$

Can you compute the limit if existent?

Exercise 4 (Complex numbers)**.** Determine and draw the following set for the values $r = \frac{1}{2}$, $r = 1$ and $r = 2$:

$$\left\{ z \in \mathbb{C} : \left| \frac{z-1}{z+1} \right| < r \right\}.$$