Fakultät MI, Fachrichtung Mathematik
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UNIVERSITÄT DES SAARLANDES

Mathematics for computer science 1

Winterterm 2019/20

Hand in your solution sheet in the mailboxes (next to Zeichensaal U.39, building E2 5) by Jan. 8 **before the lecture**.

All exercise sheets and course information can be found at: www.math.uni-sb.de/ag/schreyer/

Sheet 9 18. December 2019

Exercise 1 (Continuity)**.** The three functions $f, g, h\colon \mathbb{R} \to \mathbb{R}$ are defined as follows:

$$f(x) = \begin{cases} x, & x \in \mathbb{Q}, \\ 1-x, & x \notin \mathbb{Q}, \end{cases}$$

$$g(x) = \begin{cases} 1, & x \in \mathbb{Q}, \\ 0, & x \notin \mathbb{Q}, \end{cases}$$

$$h(x) = \begin{cases} \frac{1}{q}, & x = \frac{p}{q} \in \mathbb{Q}\backslash\{0\} \text{ with } p, q \in \mathbb{Z} \text{ coprime, } q > 0, \\ 0, & x \in \mathbb{R}\backslash\mathbb{Q} \cup \{0\}. \end{cases}$$

Show that f is only continuous in $\frac{1}{2}$, g is nowhere continuous and h is exactly continuous in all irrational x and in zero.

Exercise 2 (Continuous functions)**.**

(a) Does there exist a continuous function $f\colon \mathbb{R} \to \mathbb{R}$ that takes each of its values exactly twice?

(b) Does there exist a continuous function $f\colon \mathbb{R} \to \mathbb{R}$ that takes each of its values exactly three times?

Exercise 3 (Product rule)**.** Let $D \subset \mathbb{R}$ and let $f, g : D \to \mathbb{R}$ be two n-times differentiable functions. Show that

$$(f \cdot g)^{(n)} = \sum_{k=0}^{n} \binom{n}{k} f^{(n-k)} \cdot g^{(k)}.$$

Exercise 4 (Multiple zeros)**.** For which values $a, b \in \mathbb{R}$ exists a double zero of $f(x) = x^3 - ax + b$ (i.e., an x_0 with $f(x_0) = f'(x_0) = 0$)? For which values a, b exist exactly one, two or three real zeros of f, respectively?

We wish you a Merry Christmas and a Happy New Year!!!