Fakultät MI, Fachrichtung Mathematik
Prof. Dr. Frank-Olaf Schreyer

UNIVERSITÄT DES SAARLANDES

Computer Algebra and Gröbner Bases

Winterterm 2020/21

All exercise sheets and course information can be found at: www.math.uni-sb.de/ag/schreyer/

Sheet 1 12. November 2020

Exercise 1. Let K be an infinite field and $f \in K[x_1, \ldots, x_n]$ a non-zero polynomial. Show that there exists a point $a \in \mathbb{A}^n(K)$ such that $f(a) \neq 0$.

Exercise 2. Let $K[x]$ a polynomial ring in one variable over a field. Prove that $K[x]$ is a principal ideal domain, that is, every ideal $I \subset K[x]$ is generated by a single polynomial.

Exercise 3. Let $M, N \subset L$ be two submodules of an R-module L. Prove:

(a) $M + N = \{n + m \in L \mid m \in M \text{ and } n \in N\}$ and $M \cap N$ are submodules of L.
(b) $(M + N)/M \cong N/(N \cap M)$.

Exercise 4. Let $M \subset N \subset L$ be a chain of submodules of an R-module L. Prove:

(a) N/M is a submodule of L/M.
(b) $(L/M)/(N/M) \cong L/N$.