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Computer Algebra and Gröbner Bases

Winterterm 2020/21

All exercise sheets and course information can be found at: www.math.uni-sb.de/ag/schreyer/

Sheet 3 3. December 2020

Exercise 1. Consider $R = C^0[0,1] = \{f : [0,1] \to \mathbb{R} \mid f \text{ is continuous}\}$, the ring of continuous function on the interval $[0,1]$. Prove that R is neither an integral domain nor noetherian.

Exercise 2.

(1) Let $R = K[A]$ be the coordinate ring of a variety and $f \in R$ be an element which is not a unit. Prove that R_f is isomorphic to the coordinate ring of a variety as well.
(2) Prove that $K[x]_{(x)}$, the localization of the polynomial ring in one variable at the maximal ideal (x), is not isomorphic to the coordinate ring of a variety.

Exercise 3. Let $I, J \subset R$ be ideals in a ring, and let $U \subset R$ be a multiplicative subset. Prove

$$(I : J)[U^{-1}] = I[U^{-1}] : J[U^{-1}].$$

Exercise 4. Let $A = B \cup C$ be a decomposition of an algebraic set into proper algebraic subsets. Let $p \in A \setminus C$ be a point and $\mathfrak{m} = \mathrm{I}(p)$ be the corresponding maximal ideal. Prove

$$K[A]_{\mathfrak{m}} \cong K[B]_{\mathfrak{m}}.$$