Fakultät MI, Fachrichtung Mathematik
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Computer Algebra and Gröbner Bases

Winterterm 2020/21

All exercise sheets and course information can be found at: www.math.uni-sb.de/ag/schreyer/

Sheet 8 14. January 2021

Exercise 1. Let $>$ be monomial order and let $M \subset K[x_1, \ldots, x_n]$ be a finite set of monomials. Prove that there exiss a weight order $>_w$ (with $\mathbb{Q}$-linearly independent weights) which induces the same order on the monomials of M as $>$.

Hint: Consider the convex hull C of the set

$$\{\alpha - \beta \mid x^\alpha, x^\beta \in M \text{ with } x^\alpha \geq x^\beta\} \subset \mathbb{R}^n$$

and prove that $0 \in C$ is a vertex of C, i.e., 0 is not a linear combination of other points in C with strictly positive coefficients.

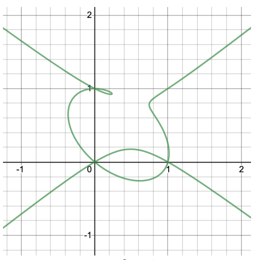
Exercise 2. Prove that the Segre product $\mathbb{P}^n \times \mathbb{P}^m \subset \mathbb{P}^N$ with $N = (n+1)(m+1) - 1$ has dimension $\dim \mathbb{P}^n \times \mathbb{P}^m = n + m$ and degree $\deg \mathbb{P}^n \times \mathbb{P}^m = \binom{n+m}{n}$.

Exercise 3. Give examples of two smooth plane conics which intersect in points with multiplicities

(a) $1, 1, 1, 1$
(b) $2, 1, 1$
(c) $2, 2$
(d) $3, 1$
(e) 4

Exercise 4. Use Macaulay2 to compute

(a) a rational parametrization of the plane quartic curve $V(f) \subset \mathbb{A}^2$ defined by $f = -2\,x^4 - 2\,x^3y + x^2y^2 + 3\,x\,y^3 + 4\,y^4 + 4\,x^3 + x^2y - 4\,x\,y^2 - 8\,y^3 - 2\,x^2 + x\,y + 4\,y^2$. Hint: $V(f)$ contains the points with coordinates $(0, 0), (1, 0), (0, 1)$ and $(1, 1)$.



(b) the equation of the image C of

$$\varphi : \mathbb{P}^1 \to \mathbb{P}^2, [t_0 : t_1] \mapsto [t_0^4 : t_0^3t_1 - t_0t_1^3 : t_1^4].$$

Where are the singular points of C?