UNIVERSITÄT DES SAARLANDES
Fachrichtung Mathematik
Prof. Dr. R. Schulze-Pillot

UNIVERSITÄT DES SAARLANDES

Lecture on modular forms and modular curves

Winter semester 2017/18, Mi 14-16 (starting Oct. 18), Fr 10-12 every second week (starting October 20), SR 10, E2 4.
Lecture times are negotiable if there are collisions, please let me know.

The subject of this course is closely connected to the seminar *Topics on elliptic curves* of Prof. Weitze-Schmithüsen in this semester.
Modular functions for a subgroup $\Gamma \subseteq SL_2(\mathbb{Z})$ of finite index are meromorphic functions on the complex upper half plane which are invariant under the fractional linear transformations $z \mapsto \frac{az+b}{cz+d}$ with $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \Gamma$, they can also be viewed as functions on the modular curve H/Γ obtained as the quotient of the upper half plane H by this group action. These curves are called modular because of their connection to moduli of elliptic curves with additional data and certain other moduli problems.
Modular forms are holomorphic functions on H which transform under a group Γ as above according to $f(\frac{az+b}{cz+d}) = (cz+d)^k f(z)$ for some integer k and satisfy a growth condition when z approaches the boundary of H. For fixed Γ and k these functions form a finite dimensional complex vector space (this is not trivial but one of the first results of the theory). They were first studied as a tool for embedding a modular curve into projective space but soon gained number theoretic significance in their own right, for example they played a key role in Andrew Wiles' proof of Fermat's last theorem. More general, according to a set of conjectures of Langlands, the structure of many objects of arithmetic algebraic geometry should be connected to the properties of generalizations of modular forms to subgrups of general linear, orthogonal and symplectic groups; such connections have meanwhile been established in many cases but not in the full generality in which they have been predicted. Their study continues to be one of the most active fields of number theoretic research.

The course will first treat the action of discrete subgroups of $SL_2(\mathbb{R})$ on the upper half plane and fundamental domains and the complex structure of the quotient of H by such a group action. We will then study the basic theory of modular forms, in particular the connection to functions on the group $SL_2(\mathbb{R})$, Hecke operators, L-functions.

The course will be given in English if there are participants who don't understand lectures given in German, in German otherwise.

Prerequisites: Prerequisites are a course in complex analysis (Funktionentheorie) and some elementary number theory.

Literature:

- Diamond, Shurman: A first course in modular forms
- Koecher, Krieg: Elliptische Funktionen und Modulformen
- Miyake: Modular forms
- Shimura: Introduction to the arithmetic theory of automorphic functions