UNIVERSITÄT DES SAARLANDES
Fakultät für Mathematik und Informatik
Fachrichtung Mathematik

Mathematisches Kolloquium

Am Freitag, dem 19. Mai 2017 spricht um 14 Uhr c. t. im Hörsaal IV der Fachrichtung Mathematik (Gebäude E2 4)

Prof. Dr. Roland Duduchava

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über das Thema:

Mixed Boundary Value Problems for the Laplace Beltrami Equation

Abstract: Let $\mathscr{C}$ be a smooth hypersurface in $\mathbb{R}^3$ with a smooth boundary decomposed into two connected $\partial\mathscr{C} = \Gamma = \Gamma_D \cup \Gamma_N$ and non-intersecting $\Gamma_D \cap \Gamma_N = \emptyset$ parts. Let $\nu(\omega) = (\nu_1(\omega), \nu_2(\omega), \nu_3(\omega))$, $\omega \in \overline{\mathscr{C}}$ be the unit normal vector field on the surface $\mathscr{C}$. Let us consider the Laplace-Beltrami operator written in terms of the Günter's tangent derivatives $\Delta_{\mathscr{C}} := \mathscr{D}_1^2 + \mathscr{D}_2^2 + \mathscr{D}_3^2$, $\quad \mathscr{D}_j := \partial_j - \nu_j \partial_\nu$, $j = 1, 2, 3$, $\partial_\nu = \sum_{j=1}^3 \nu_j \partial_j$. Let $\nu_\Gamma(t) = (\nu_{\Gamma,1}(t), \nu_{\Gamma,2}(t), \nu_{\Gamma,3}(t))$, $t \in \Gamma$, be the unit normal vector field on the boundary Γ, which is tangential to the surface $\mathscr{C}$ and directed outside of the surface. We study the following mixed boundary value problem for the Laplace-Beltrami equation

$$\begin{cases} \Delta_{\mathscr{C}} u(t) = f(t), & t \in \mathscr{C}, \\ u^+(\tau) = g(\tau), & \tau \in \Gamma_D, \\ (\partial_{\nu_\Gamma} u)^+(\tau) = h(\tau), & \tau \in \Gamma_N, \\ \qquad \partial_{\nu_\Gamma} := \sum_{j=1}^3 \nu_{\Gamma,j} \mathscr{D}_j. \end{cases} \tag{1}$$

Lax-Milgram Lemma applied to the BVP (1) gives that the BVP (1) has a unique solution in the classical setting $f \in \widetilde{\mathbb{H}}^{-1}(\mathscr{C})$, $\quad g \in \mathbb{H}^{1/2}(\Gamma)$, $\quad h \in \mathbb{H}^{-1/2}(\Gamma)$. But in some problems, for example in approximation methods, it is important to know the solvability properties in the non-classical setting

$$f \in \widetilde{\mathbb{H}}_p^{s-2}(\mathscr{C}), \quad g \in \mathbb{W}_p^{s-1/p}(\Gamma), \\ h \in \mathbb{W}_p^{s-1-1/p}(\Gamma), \quad 1 < p < \infty, \quad s > \frac{1}{p}. \tag{2}$$

To this end we prove the following.

THEOREM. *Let $1 < p < \infty$, $s > \dfrac{1}{p}$. The BVP (1) is not Fredholm in the non-classical setting (2) if and only if:*

$$\begin{cases} p \neq \frac{4}{3}, 2, 4 \quad \text{and} \quad s > \frac{1}{p} \quad \text{is arbitrary}, \\ p = \frac{4}{3}, \quad s \neq \frac{1}{p} + \frac{1}{4} + k, \quad k = -1, 0, 1, \ldots \\ p = 2 \quad \text{and} \quad s \neq \frac{1}{p} + k, \quad k = 0, 1, \ldots, \\ p = 4, \quad s \neq \frac{1}{p} + \frac{3}{4} + k, \quad k = -1, 0, 1, \ldots. \end{cases}$$

In particular, the BVP (1) has a unique solution u in the non-classical setting (2) if

$$\frac{4}{3} < p < 4 \quad \text{and} \quad \frac{1}{p} + \frac{1}{4} < s < \frac{1}{p} + \frac{3}{4}.$$

Der Gast wird von Prof. Dr. Sergej Rjasanow betreut.

Alle Interessenten sind zum Vortrag herzlich eingeladen.

Kaffee und Tee ab 13.45 Uhr im Didaktiklabor der Mathematik (Erdgeschoss, Raum 114)

Die Dozenten der Mathematik