UNIVERSITÄT DES SAARLANDES FACHRICHTUNG 6.1 – MATHEMATIK Prof. Dr. Roland Speicher M.Sc. Felix Leid



## Free Probability

Course in the Winter Term 2018/2019

Free probability is a quite young mathematical theory with many avatars. It started in the theory of operator algebras, showed its beautiful combinatorial structure via noncrossing partitions, made contact with the world of random matrices, and reached out to many other subjects like representation theory of large groups, quantum groups, invariant subspace problem, large deviations, quantum information theory, subfactors, or statistical inference. Even in physics and engineering, people have heard of it and find it useful and exciting.

This course is in a sense a continuation of both the classes "Random Matrices" and "Operator Algebras" from the summer term 2018, as it deals with the concept of "freeness" or "free independence", which appears both in the limit of random matrices as well as for important classes of von Neumann algebras. On the other hand, freeness is an important concept for its own sake which deserves to be investigated independently of its random matrix or operator algebraic roots. That's what we will do in this course; we will, in particular, look at the combinatorial, analytical and probabilistic structure of freeness.

$$\varphi(a_1b_1a_2b_2) = \varphi(a_1a_2)\varphi(b_1)\varphi(b_2) + \varphi(a_1)\varphi(a_2)\varphi(b_1b_2) - \varphi(a_1)\varphi(a_2)\varphi(b_1)\varphi(b_2)$$

$$a_1 \ b_1 \ a_2 \ b_2 \ a_1 \ b_1 \ a_2 \ b_2 \ a_1 \ b_1 \ a_2 \ b_2 \ a_1 \ b_1 \ a_2 \ b_2$$

Figure 1: Combinatorial rule computing mixed moments of free random variables

Its relations to random matrices and to operator algebras (and in particular its use in those contexts) will also be covered, however, depending on the audience, we will recall the relevant basic knowledge from those subjects; i.e., having taken an operator algebra and/or random matrix course is surely helpful, but not required for this course.

## Time: Monday, 10 - 12 and Wednesday, 10 - 12Place: to be announced

This course consists of two lectures and a tutorial on a weekly basis, by successful participation students can acquire 9 credit points. There is a possibility to set up topics for bachelor- and master theses based on the course. For further information contact Felix Leid (leid@math.uni-sb.de) or see

 $www.math.uni-sb.de/ag/speicher\_lehre\_FreeProbwise1819$