Random Matrices and Free Convolution

Roland Speicher

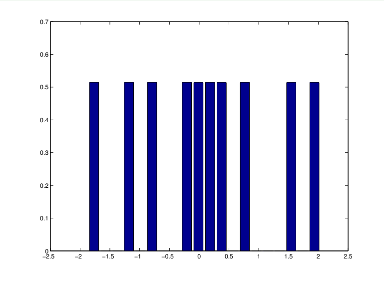
Saarland University
Saarbrücken, Germany

January 23, 2019

Wigner and Voiculescu

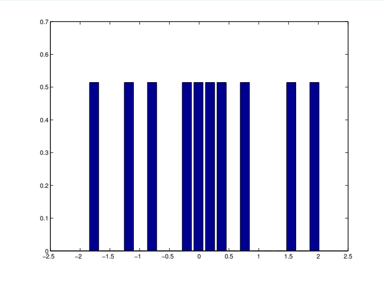
Wigner's semicircle law (Wigner 1955)

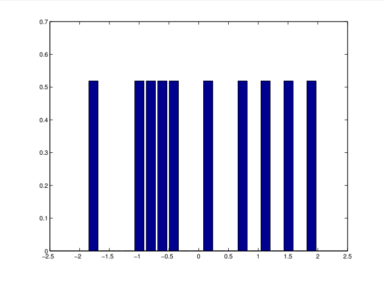
10 eigenvalues of GUE(10)



Wigner's semicircle law (Wigner 1955)

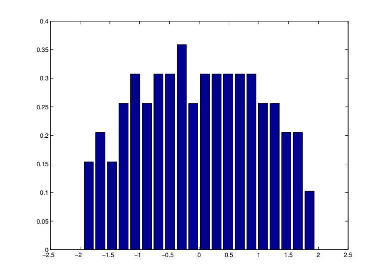
10 eigenvalues of GUE(10)

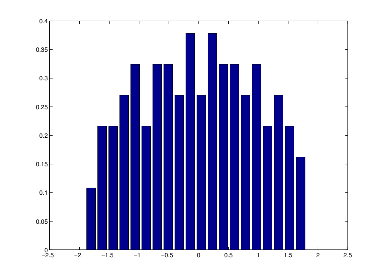




Wigner's semicircle law (Wigner 1955)

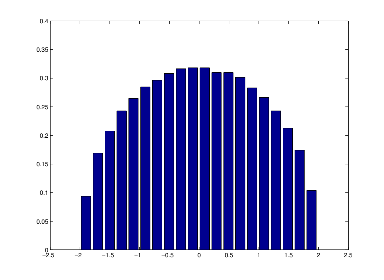
100 eigenvalues of GUE(100)

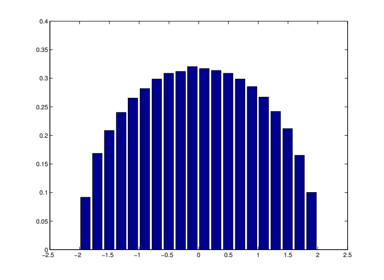




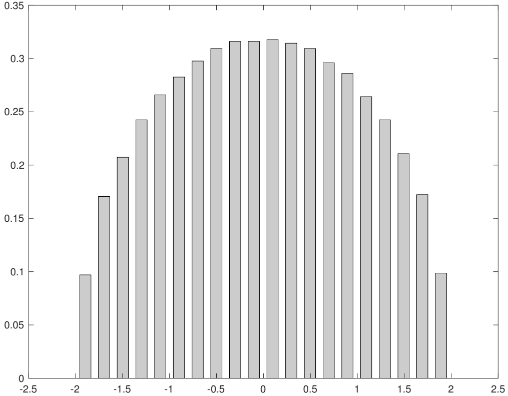
Wigner's semicircle law (Wigner 1955)

3000 eigenvalues of GUE(3000)



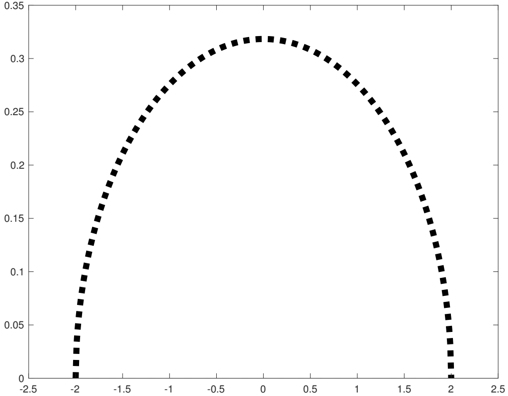


One-matrix case: Wigner's semicircle law



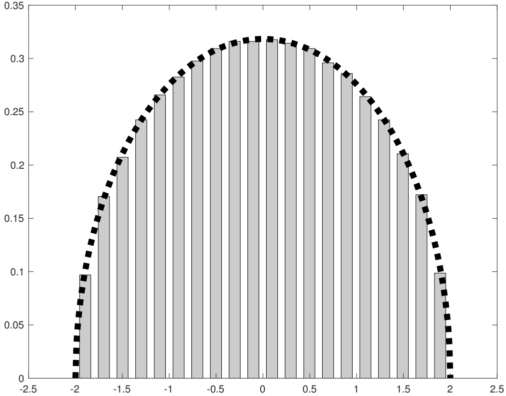
- eigenvalues of GUE(3000)

One-matrix case: Wigner's semicircle law



- semicircular distribution

One-matrix case: Wigner's semicircle law



- eigenvalues of GUE(3000)
- semicircular distribution

GUE + deterministic (diagonal) matrix

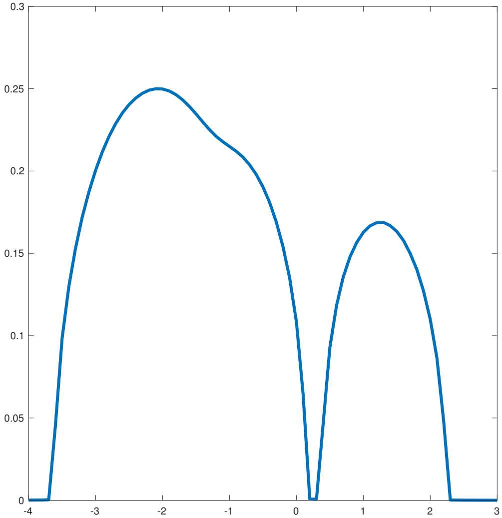
- $A_N + D_N$
  A_N GUE(N)
  $D_N = \text{diag}(-2, -2, -1, 1)$

GUE + deterministic (diagonal) matrix

- $A_N + D_N$
  A_N GUE(N)
  $D_N = \text{diag}(-2, -2, -1, 1)$

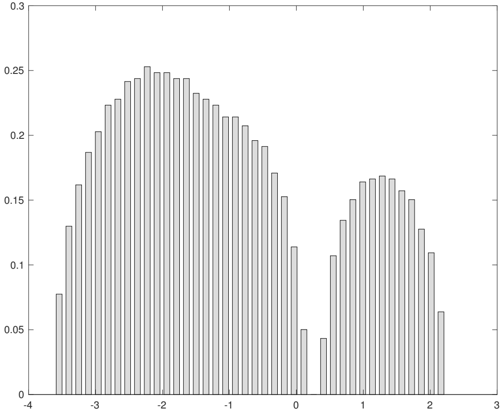
- $\mu_d = \frac{1}{4}(2\delta_{-2} + \delta_{-1} + \delta_{+1})$

GUE + deterministic (diagonal) matrix



- $\mu_d = \frac{1}{4}(2\delta_{-2} + \delta_{-1} + \delta_{+1})$
- $\mu_s \boxplus \mu_d$
  via subordination iteration

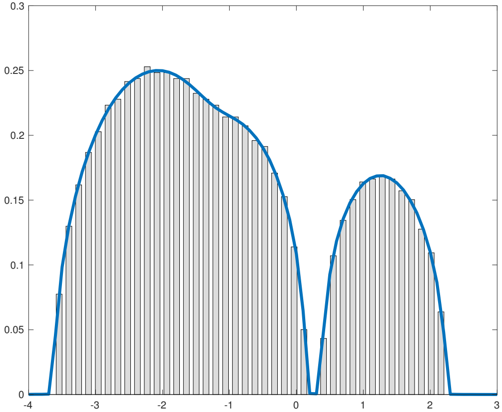
GUE + deterministic (diagonal) matrix



- $A_N + D_N$
  A_N GUE(N)
  $D_N = \text{diag}(-2, -2, -1, 1)$

- eigenvalues of $A_N + D_N$,
  for $N = 3000$

GUE + deterministic (diagonal) matrix



- $A_N + D_N$
  A_N GUE(N)
  $D_N = \text{diag}(-2, -2, -1, 1)$
- $\mu_d = \frac{1}{4}(2\delta_{-2} + \delta_{-1} + \delta_{+1})$
- $\mu_s \boxplus \mu_d$
  via subordination iteration
- eigenvalues of $A_N + D_N$, for $N = 3000$

sum of two randomly rotated diagonal matrices

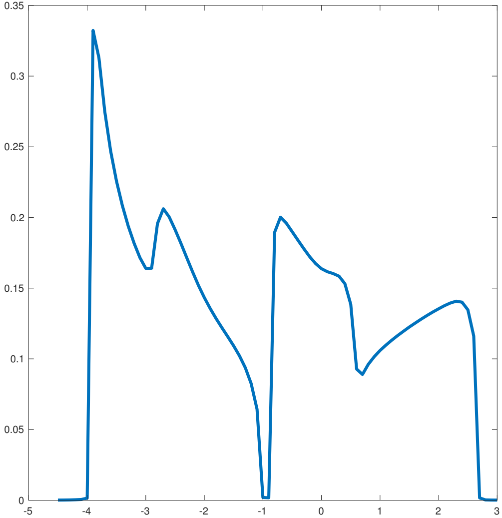
- $D_1 + UD_2U^*$
  $D_1 = \mathrm{diag}(-2, -1, 1, 2)$,
  $D_2 = \mathrm{diag}(-2, -2, -1, 1)$

sum of two randomly rotated diagonal matrices

- $D_1 + UD_2U^*$
  $D_1 = \mathsf{diag}(-2, -1, 1, 2)$,
  $D_2 = \mathsf{diag}(-2, -2, -1, 1)$

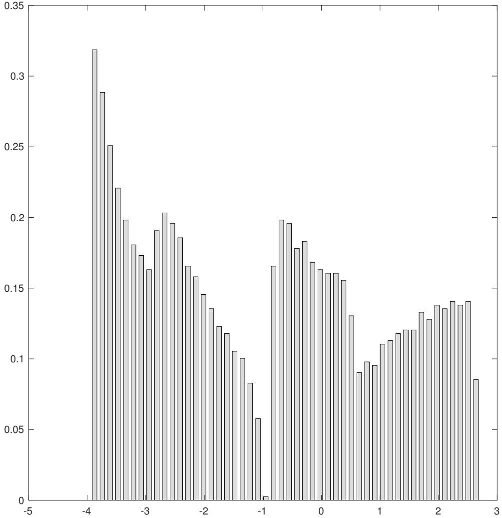
- $\mu_1 = \frac{1}{4}(\delta_{-2} + \delta_{-1} + \delta_1 + \delta_2)$
  $\mu_2 = \frac{1}{4}(2\delta_{-2} + \delta_{-1} + \delta_1)$

sum of two randomly rotated diagonal matrices



- $\mu_1 = \frac{1}{4}(\delta_{-2} + \delta_{-1} + \delta_1 + \delta_2)$
  $\mu_2 = \frac{1}{4}(2\delta_{-2} + \delta_{-1} + \delta_1)$
- $\mu_1 \boxplus \mu_2$
  via subordination iteration

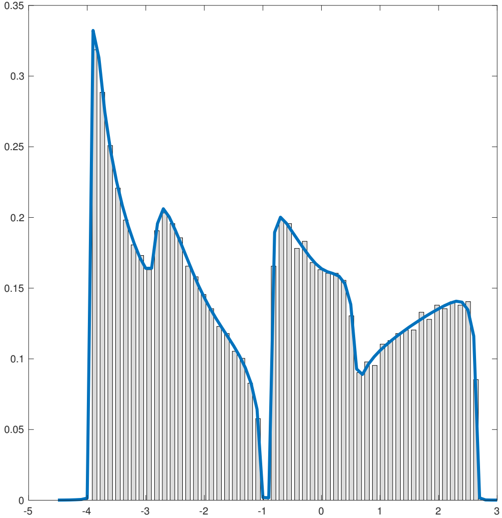
sum of two randomly rotated diagonal matrices



- $D_1 + UD_2U^*$
  $D_1 = \text{diag}(-2, -1, 1, 2)$,
  $D_2 = \text{diag}(-2, -2, -1, 1)$

- 3000 eigenvalues of
  $D_1 + UD_2U^*$

sum of two randomly rotated diagonal matrices



- $D_1 + UD_2U^*$
  $D_1 = \text{diag}(-2, -1, 1, 2)$,
  $D_2 = \text{diag}(-2, -2, -1, 1)$
- $\mu_1 = \frac{1}{4}(\delta_{-2} + \delta_{-1} + \delta_1 + \delta_2)$
  $\mu_2 = \frac{1}{4}(2\delta_{-2} + \delta_{-1} + \delta_1)$
- $\mu_1 \boxplus \mu_2$
  via subordination iteration
- 3000 eigenvalues of
  $D_1 + UD_2U^*$