Fachrichtung 6.1 – Mathematik
Wintersemester 2015/16
Jun.Prof. Johannes Rau

UNIVERSITÄT DES SAARLANDES

Refresher course for the entrance test in MINT studies
Exercise sheet 1

Exercise 1. Set

$$\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \vec{w} = \begin{pmatrix} -2 \\ 6 \\ -1 \\ 1 \end{pmatrix}.$$

a) Compute the vector $3\vec{v} - 2\vec{w} \in \mathbb{R}^4$.

b) Compute the lengths $\|\vec{v}\|$ and $\|\vec{w}\|$.

c) Rescale v to length 1 (this means find $\lambda \in \mathbb{R}$ such that $\|\lambda\vec{v}\| = 1$ and compute $\lambda\vec{v}$).

Exercise 2. Set

$$\vec{x} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \vec{y} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \vec{w} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}.$$

a) Compute the lengths $\|\vec{x}\|$, $\|\vec{y}\|$, $\|\vec{v}\|$, and $\|\vec{w}\|$.

b) Compute the scalar products $\vec{x} \cdot \vec{y}$ and $\vec{v} \cdot \vec{w}$.

c) Compute the angle between $\vec{x}$ and $\vec{y}$ and the angle between $\vec{v}$ and $\vec{w}$.

Exercise 3. For any $a \in \mathbb{R}$ consider the vectors

$$\vec{x} = \begin{pmatrix} 4 \\ a \\ 2a+1 \end{pmatrix}, \vec{y} = \begin{pmatrix} 3a \\ -8 \\ -2 \end{pmatrix}, \vec{v} = \begin{pmatrix} 1 \\ 7 \\ a+2 \\ -2 \end{pmatrix}, \vec{w} = \begin{pmatrix} 3 \\ a \\ -3 \\ a \end{pmatrix}.$$

a) For which values of a are $\vec{x}$ and $\vec{y}$ orthogonal?

b) For which values of a are $\vec{v}$ and $\vec{w}$ orthogonal?

Exercise 4. Set

$$\vec{v} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \vec{w} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}.$$

a) Compute the cross products $\vec{v} \times \vec{w}$ as well as $\vec{w} \times \vec{v}$. Compare!

b) Set $\vec{z} = \vec{v} \times \vec{w}$. Show explicitly that $\vec{z} \perp \vec{x}$ as well as $\vec{z} \perp \vec{y}$.