

Refresher course for the entrance test in MINT studies
Exercise sheet 2

Exercise 1. Solve the following systems of equations. (Use matrix notation and the Gauß algorithm. How many solutions are there?)

$$\begin{aligned}x_1 + x_2 + x_3 &= 3 \\x_1 + 2x_2 + 3x_3 &= 6\end{aligned}$$

$$\begin{aligned}x_2 - 2x_3 &= 6 \\2x_1 + 6x_2 + 4x_3 &= 8 \\x_1 + 4x_2 &= 10\end{aligned}$$

$$\begin{aligned}x_1 + x_2 + x_3 + x_4 &= 4 \\x_1 - x_2 - x_3 + x_4 &= 0 \\3x_1 - x_2 - x_3 + 3x_4 &= 2\end{aligned}$$

$$\begin{aligned}x_1 + x_3 &= 0 \\2x_1 + x_2 + 2x_4 &= 0 \\x_2 - x_4 &= 4 \\x_1 - x_2 - x_3 + x_4 &= -2 \\3x_1 + x_2 + x_3 + 2x_4 &= 0\end{aligned}$$

$$\begin{aligned}3(2x_1 - x_2) + 4(x_1 - 2x_2) &= 87 \\2(3x_1 - x_2) - 3(x_1 - x_2) &= 82\end{aligned}$$

Exercise 2. Here is a system of equations in two variables x_1, x_2 with one parameter $a \in \mathbb{R}$. Solve the system (this means describe the solutions as functions of a). How does the number of solutions depend on a ?

$$\begin{aligned}x_1 + x_2 &= a \\ax_1 - x_2 &= 1\end{aligned}$$

Exercise 3. *For fun!*

When I was born, my father was 36 years old. In 3 years, I will have half the age of my father. How old am I? (This is a system of linear equations!)