UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG MATHEMATIK
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Assignments for the lecture
Introduction to Noncommutative Differential Geometry
Summer term 2019

Assignment 1B
for the tutorial on *Tuesday, April 23, 10:15 am* (in Seminar Room 10)

Exercise 1. Let $(\mathcal{A}, \mathcal{H}, \mathcal{D})$ be a spectral triple and let $V \in B(\mathcal{H})$ be any selfadjoint operator. Prove that $(\mathcal{A}, \mathcal{H}, \mathcal{D}_V)$ for the unbounded operator $\mathcal{D}_V$ given by $\mathcal{D}_V := \mathcal{D} + V$ with domain $\operatorname{dom}(\mathcal{D}_V) := \operatorname{dom}(\mathcal{D})$ is again a spectral triple.

Exercise 2.

(i) Let $x_0 \in \mathbb{R}^n$ be given. For $j = 1, \dots, n$, we define a linear map $\partial_j|_{x_0} : C^\infty_{x_0}(\mathbb{R}^n) \to \mathbb{R}$ by $\partial_j|_{x_0}([f]_{x_0}) := (\partial_j f)(x_0) = \frac{\partial f}{\partial x_j}(x_0)$ for every germ $[f]_{x_0} \in C^\infty_{x_0}(\mathbb{R}^n)$. Prove that $\{\partial_j|_{x_0} \mid j = 1, \dots, n\}$ forms a basis of the tangent space $T_{x_0}\mathbb{R}^n$.

(ii) Let $\mathcal{M}$ be a n-dimensional smooth manifold with the maximal smooth atlas $\mathcal{A} = \{(U_i, \varphi_i) \mid i \in I\}$. Show that for every $i \in I$ and each $x_0 \in U_i$, the linear map

$$\Theta_{i,x_0} :\ \mathbb{R}^n \to T_{x_0}\mathcal{M}$$

that is defined by

$$\Theta_{i,x_0}(v)\big([f]_{x_0}\big) := \sum_{j=1}^n v_j \big(\partial_j(f \circ \varphi_i^{-1})\big)(\varphi_i(x_0))$$

for each $v = (v_1, \dots, v_n) \in \mathbb{R}^n$ and every germ $[f]_{x_0} \in C^\infty_{x_0}(\mathcal{M})$, is an isomorphism of real vector spaces.