UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG MATHEMATIK
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Assignments for the lecture
Introduction to Noncommutative Differential Geometry
Summer term 2019

Assignment 2B
for the tutorial on *Tuesday, May 7, 10:15 am* (in Seminar Room 10)

Exercise 1. Complete the proof of Theorem 2.9 of the lecture by proving the following assertions for a smooth manifold $\mathcal{M}$ of dimension n and an open subset $V \subseteq \mathcal{M}$:

(i) For every $D \in \operatorname{der} C^\infty(V)$, the map $\Psi(D) : V \to T\mathcal{M}, x \mapsto (\Psi(D))(x)$ belongs to $\mathfrak{X}(V)$. Recall that $(\Psi(D))(x_0) \in T_{x_0}\mathcal{M}$ for any point $x_0 \in V$ is defined by

$$(\Psi(D))(x_0) : \; C^\infty_{x_0}(\mathcal{M}) \to \mathbb{R}, \quad [f]_{x_0} \mapsto D|_{x_0}(\rho \cdot f|_V),$$

where $\rho : \mathcal{M} \to [0,1]$, for a chosen representative (U, f) of the given germ $[f]_{x_0}$, is a bump function for (U, x_0).

(ii) The induced linear map $\Psi : \operatorname{der} C^\infty(V) \to \mathfrak{X}(V), D \mapsto \Psi(D)$ satisfies

$$\Phi \circ \Psi = \operatorname{id}_{\operatorname{der} C^\infty(V)} \qquad \text{and} \qquad \Psi \circ \Phi = \operatorname{id}_{\mathfrak{X}(V)},$$

where $\Phi : \mathfrak{X}(V) \to \operatorname{der} C^\infty(V)$ is the linear map defined in Theorem 2.9.