UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG MATHEMATIK
Dr. Tobias Mai

Assignments for the lecture
Introduction to Noncommutative Differential Geometry
Summer term 2019

Assignment 5A
for the tutorial on *Tuesday, June 25 (!), 10:15 am* (in Seminar Room 10)

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*Due to a public holiday, there will be **no lecture** on Monday, June 10; instead, we will have an **additional lecture** on Tuesday, June 11, at the time of the problem sessions. Note that there will be also **no lecture** on Monday, June 17, and **no problem session** on Tuesday, June 18; the **next problem session** is thus postponed to Tuesday, June 25.*

Exercise 1. Let $\mathcal{H}$ be a separable complex Hilbert space of infinite dimension. Prove for every $T \in K(\mathcal{H})$ and each $N \in \mathbb{N}$ that

$$\sigma_N(T) = \sup \left\{ \|TP\|_1 \;\middle|\; P \in B(\mathcal{H}) \text{ orthogonal projection with } \dim(P\mathcal{H}) = N \right\}.$$

Exercise 2. Let $\mathcal{H}$ be a separable complex Hilbert space of infinite dimension. Prove the following statements, which were made in Proposition 4.7 of the lecture:

(i) For $T_1, T_2 \in K(\mathcal{H})$ and each $N \in \mathbb{N}$, we have that

$$\sigma_N(T_1 + T_2) \leq \sigma_N(T_1) + \sigma_N(T_2).$$

Conclude that σ_N for each $N \in \mathbb{N}$ is a norm on $K(\mathcal{H})$.

(ii) For positive $T_1, T_2 \in K(\mathcal{H})$ and each $N \in \mathbb{N}$, we have that

$$\sigma_{2N}(T_1 + T_2) \geq \sigma_N(T_1) + \sigma_N(T_2).$$

(iii) For positive $T_1, T_2 \in K(\mathcal{H})$ and each $N \in \mathbb{N}$, we have that

$$\gamma_N(T_1 + T_2) \leq \gamma_N(T_1) + \gamma_N(T_2) \leq \gamma_{2N}(T_1 + T_2) \left(1 + \frac{\log(2)}{\log(N)}\right).$$