Due to a public holiday, there will be no lecture on Monday, June 10; instead, we will have an additional lecture on Tuesday, June 11, at the time of the problem sessions. Note that there will be also no lecture on Monday, June 17, and no problem session on Tuesday, June 18; the next problem session is thus postponed to Tuesday, June 25.

Exercise 1. Let $\mathcal{H}$ be a separable complex Hilbert space of infinite dimension. Prove for every $T \in K(\mathcal{H})$ and each $N \in \mathbb{N}$ that

$$
\sigma_N(T) = \sup \{ \|TP\|_1 \mid P \in B(\mathcal{H}) \text{ orthogonal projection with } \dim(P\mathcal{H}) = N \}.
$$

Exercise 2. Let $\mathcal{H}$ be a separable complex Hilbert space of infinite dimension. Prove the following statements, which were made in Proposition 4.7 of the lecture:

(i) For $T_1, T_2 \in K(\mathcal{H})$ and each $N \in \mathbb{N}$, we have that

$$
\sigma_N(T_1 + T_2) \leq \sigma_N(T_1) + \sigma_N(T_2).
$$

Conclude that $\sigma_N$ for each $N \in \mathbb{N}$ is a norm on $K(\mathcal{H})$.

(ii) For positive $T_1, T_2 \in K(\mathcal{H})$ and each $N \in \mathbb{N}$, we have that

$$
\sigma_{2N}(T_1 + T_2) \geq \sigma_N(T_1) + \sigma_N(T_2).
$$

(iii) For positive $T_1, T_2 \in K(\mathcal{H})$ and each $N \in \mathbb{N}$, we have that

$$
\gamma_N(T_1 + T_2) \leq \gamma_N(T_1) + \gamma_N(T_2) \leq \gamma_{2N}(T_1 + T_2) \left(1 + \frac{\log(2)}{\log(N)}\right).
$$