



Assignments for the lecture
Introduction to Noncommutative Differential Geometry
Summer term 2019

Assignment 6B
for the tutorial on *Tuesday, July 9, 10:15 am* (in Seminar Room 10)

Exercise 1. Let $(\mathcal{A}, \mathcal{H}, \mathcal{D})$ be a spectral triple which is p -summable for some $1 \leq p < \infty$. Prove that $(\mathcal{A}, \mathcal{H}, \mathcal{D})$ is also θ -summable; more precisely, show that for each $t_0 > 0$ a constant $C > 0$ exists such that

$$\mathrm{Tr}(e^{-t\mathcal{D}^2}) \leq Ct^{-p/2} \quad \text{for all } 0 < t < t_0.$$

Hint: Use the unbounded functional calculus for \mathcal{D} .

Exercise 2. Let \mathcal{H} be any separable complex Hilbert space of infinite dimension. Prove that every Dixmier trace $\mathrm{Tr}_\omega : \mathcal{L}^{(1,\infty)}(\mathcal{H}) \rightarrow \mathbb{C}$ induces a seminorm by

$$\|\cdot\| : \mathcal{L}^{(1,\infty)}(\mathcal{H}) \rightarrow [0, \infty), \quad T \mapsto \mathrm{Tr}_\omega(|T|).$$