



Non-commutative Distributions

Lecture in the Summer Term 2019

The lectures will present some of the main progress which we made in recent years concerning our investigations of non-commutative distributions of non-commuting operators or random matrices. In particular, a basic problem is to find a good approach to the meaning of “non-commutative distribution”. In particular, the lectures will cover:

- (i) the operator-valued version of free probability theory (combinatorial but also analytic aspects);
- (ii) the linearization trick to reduce non-linear scalar problems to linear operator-valued problems;
- (iii) the combination of (i) and (ii) to calculate the distribution of polynomials in free variables;
- (iv) the basic theory of non-commutative rational functions (a.k. free skew field) and the extension of (iii) to such rational functions of free variables;
- (v) free analysis, which is a version of complex analysis for several non-commuting variables;
- (vi) recent results concerning the regularity of functions in non-commuting variables;
- (vii) more cool stuff, which still has to be determined.

In some sense this is a continuation of my class “Free Probability Theory” from last term. On the other hand, I will develop the theory of free probability theory again, but in a more general, operator-valued context. So, in principle and with some additional efforts, it should be possible to take the class without having a prior knowledge on free probability. Big parts of the material do also not deal so much with free variables, but more general with analytic and algebraic aspects of maximal non-commuting variables.

In any case, complex analysis and functional analysis are necessary as prerequisites.

Time and Place: Mondays and Fridays, 12–14, HS IV (E2 4)

More info and updates about the class can be found on my blog

<https://rolandspeicher.com/category/lectures/non-commutative-distributions-2019/>

The lectures will be recorded and the videos will be uploaded on our video platform. See also:

<https://www.math.uni-sb.de/ag/speicher/lehre.html>