UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG MATHEMATIK
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Assignments for the lecture on
Non-Commutative Distributions
Summer Term 2019

Assignment 5
Hand in on Friday, 31.05.19, before the lecture.

Exercise 1 (10 points).
Show the following easy direction of Theorem 4.9: Let $(\mathcal{A}, \mathcal{B}, E)$ be an operator-valued C^*-probability space and $X = X^* \in \mathcal{A}$. Show that $\mu_X \in \Sigma_{\mathcal{B}}^0$.

Exercise 2 (10 points).
Let $\Phi : \mathcal{A} \to \mathcal{B}$ be a completely positive map between two C^*-algebras with $\Phi(1) = 1$. Show that Φ satisfies the following kind of Cauchy-Schwarz inequality: for $a \in \mathcal{A}$ we have $\Phi(a)^*\Phi(a) \leq \Phi(a^*a)$.
Hint: Consider the positive matrix

$$\begin{pmatrix} a^*a & a^* \\ a & 1 \end{pmatrix}$$

Exercise 3 (10 points).
Let X and Y be free in an operator-valued probability space $(\mathcal{A}, \mathcal{B}, E)$. Calculate the mixed moment $E[Xb_1Yb_2Xb_3Y]$, for $b_1, b_2, b_3 \in \mathcal{B}$, in terms of moments of X and of Y.