UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG MATHEMATIK
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Assignments for the lecture
Potential Theory in the Complex Plane
Summer term 2020

Assignment 1 A
for the first tutorial on *Monday, May 18, 1:00 pm*

Exercise 1. Let $\emptyset \neq \Omega \subseteq \mathbb{R}^N$ be an open subset and suppose that $\phi : \mathbb{R}^N \to \mathbb{R}^N$ is

(i) an *isometry*, i.e., ϕ is of the form $\phi(x) = a + Qx$ for an orthogonal matrix $Q \in M_N(\mathbb{R})$ and a vector $a \in \mathbb{R}^N$, or

(ii) a *dilation*, i.e., ϕ is of the form $\phi(x) = \alpha x$ for some real number $\alpha > 0$.

Prove that the following statement holds true in each of those cases (see Remark 2.2 (ii) of the lecture): for every $f \in H(\phi(\Omega))$, we have that $f \circ \phi \in H(\Omega)$.

Exercise 2. Let $\emptyset \neq \Omega \subseteq \mathbb{R}^N$ be an open subset. Consider two functions $f, g \in H(\Omega)$. Show that the pointwise product $f \cdot g$ is harmonic if and only if f and g have orthogonal gradients in the sense that

$$\langle \operatorname{grad} f(x), \operatorname{grad} g(x) \rangle = 0 \qquad \text{for all } x \in \Omega.$$