UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG MATHEMATIK
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Assignments for the lecture
Potential Theory in the Complex Plane
Summer term 2020

Assignment 4 A
for the tutorial on *Monday, July 6, 1:00 pm*

Exercise 1. Let $\emptyset \neq \Omega_1, \Omega_2 \subseteq \mathbb{C}$ be open. Consider a function $f \in \mathcal{O}(\Omega_1)$ with the property that $f(\Omega_1) \subseteq \Omega_2$ and a function $s \in C^2(\Omega_2)$. Show that for each $z \in \Omega_1$

$$\big(\Delta(s \circ f)\big)(z) = (\Delta s)(f(z))\,|f'(z)|^2.$$

Exercise 2. Let T be a compact topological space and let $\emptyset \neq \Omega \subseteq \mathbb{R}^N$ be open. Suppose that $f : \Omega \times T \to [-\infty, +\infty)$ is a function with the following properties:

- f is upper semicontinuous on $\Omega \times T$;
- the function
$$f(\cdot, t) : \ \Omega \longrightarrow [-\infty, +\infty), \quad x \longmapsto f(x, t)$$
is subharmonic on Ω for each $t \in T$.

Prove that we obtain a well-defined subharmonic function $s : \Omega \to [-\infty, +\infty)$ by

$$s(x) := \sup_{t \in T} f(x, t) \qquad \text{for } x \in \Omega.$$