Assignments for the lecture
*Potential Theory in the Complex Plane*
Summer term 2020

**Assignment 4 A**
for the tutorial on *Monday, July 6, 1:00 pm*

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**Exercise 1.** Let \( \emptyset \neq \Omega_1, \Omega_2 \subseteq \mathbb{C} \) be open. Consider a function \( f \in \mathcal{O}(\Omega_1) \) with the property that \( f(\Omega_1) \subseteq \Omega_2 \) and a function \( s \in C^2(\Omega_2) \). Show that for each \( z \in \Omega_1 \)

\[
(\Delta(s \circ f))(z) = (\Delta s)(f(z)) |f'(z)|^2.
\]

**Exercise 2.** Let \( T \) be a compact topological space and let \( \emptyset \neq \Omega \subseteq \mathbb{R}^N \) be open. Suppose that \( f : \Omega \times T \to [-\infty, +\infty) \) is a function with the following properties:

- \( f \) is upper semicontinuous on \( \Omega \times T \);
- the function \( f(\cdot, t) : \Omega \to [-\infty, +\infty), \quad x \mapsto f(x, t) \)
  is subharmonic on \( \Omega \) for each \( t \in T \).

Prove that we obtain a well-defined subharmonic function \( s : \Omega \to [-\infty, +\infty) \) by

\[
s(x) := \sup_{t \in T} f(x, t) \quad \text{for } x \in \Omega.
\]