The conjecture:
On a projective non-singular algebraic variety over \(\mathbb{C}\), any Hodge class is a rational linear combination of classes \(cl(Z)\) of algebraic cycles.

1 Definition. A complex manifold is a topological space \(M\) with an atlas \((U_i)_{i \in I}\) of open sets in \(M\) covering \(M\) and a set of coordinate charts \(\Phi_i : U_i \to \mathbb{C}^n\) such that the transition maps \(g_{i,j} = \Phi_i \circ \Phi_j^{-1} : \Phi_j(U_i \cap U_j) \to \Phi_i(U_i \cap U_j)\) are biholomorphic.

2 Definition. Let \(M\) be a complex manifold. A differential form of type \((p, q)\) is a differential form that can be written as
\[
\omega = \sum_{|I|=p} \sum_{|J|=q} \varphi_{I,J} dz_I d\bar{z}_J
\]
The space of the \((p, q)\)-forms on \(M\) is denoted by \(A^{p,q}(M)\).

3 Remark. The exterior derivative operator \(d\) can be decomposed as \(d = \partial + \bar{\partial}\), where \(\partial : A^{p,q}(M) \to A^{p+1,q}(M)\) and \(\bar{\partial} : A^{p,q}(M) \to A^{p,q+1}(M)\) are the so-called Dolbeault operators.

The Dolbeault complex is the complex
\[
0 \to A^{p,0}(M) \xrightarrow{\partial} A^{p,1}(M) \xrightarrow{\partial} \ldots \xrightarrow{\partial} A^{p,n}(M) \to 0
\]
and the \(q\)-th cohomology class from this complex is called the Dolbeault cohomology class:
\[
H^{p,q}(M) = \ker \bar{\partial} : A^{p,q}(M) \to A^{p,q+1}(M)
\]

4 Proposition. Let \(M\) be a compact Kähler manifold. Then, the Hodge decomposition holds, i.e. there exists an isomorphism
\[
\bigoplus_{p+q=n} H^{p,q}(M) \xrightarrow{\cong} H^n(M, \mathbb{C}) = H^n(M, \mathbb{R}) \otimes \mathbb{C}
\]
where \(H^n(M, \mathbb{R})\) is the \(n\)-th de Rham cohomology class of \(M\).

5 Definition. A Hodge cycle is an element of the cohomology class \(H^{2p}(M, \mathbb{Q})\) which under the Hodge decomposition corresponds to an element of the Dolbeault cohomology class \(H^{p,p}(M)\).

A Hodge class is the cohomology class of such Hodge cycles: \(H^{p,p}(M) = H^{2p}(M, \mathbb{Q}) \cap H^{p,p}(M)\).

6 Definition. A (closed) projective set \(X \subseteq \mathbb{P}^n\) is a subset of the projective space, that can be written as \(X = V(S) = \{x \in \mathbb{P}^n | f(x) = 0 \forall f \in S\}\) for \(S\) a subset of the homogeneous polynomial functions \(\mathbb{C}[X_0, X_1, \ldots, X_n]\).

7 Definition. For a projective set \(X\) an open subset is a set of the form \(X \setminus Y\) where \(Y\) is a projective set in the same projective space. These open subsets define the Zariski topology on \(X\).

8 Definition. A projective variety is a projective set together with the Zariski topology and the ring of regular functions.

9 Definition. An irreducible variety \(X\) is nonsingular at \(x \in X\) if the dimension of the (embedded) tangent space at \(x\) equals the dimension of \(X\): \(\dim T_x X = \dim X\), else singular.

\(X\) is called nonsingular if it is nonsingular at all points.

10 Definition. An algebraic cycle of an algebraic variety \(X\) is a formal linear combination \(V = \sum c_i Z_i\) of irreducible, closed subvarieties \(Z_i\).