UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG MATHEMATIK
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Random Matrices

Summer term 2018

Assignment 9

Due: Tuesday, June 26, 2018, before the lecture
Hand in your solution at the beginning of the lecture or drop it into letterbox 47.

Problem 28 (10 + 10* points)**.** (You can work on this exercise for two weeks, so it is due on Tuesday, July 3.) Produce histograms for the Tracy-Widom distribution. For this, plot $(\lambda_{\max} - 2)N^{\frac{2}{3}}$ and do the following:

(i) Produce histograms for the largest eigenvalue of GUE(N). Do this for $N = 50$, $N = 100$, $N = 200$ with at least 5000 trials in each case.

(ii) Produce histograms for the largest eigenvalue of GOE(N). Do this for $N = 50$, $N = 100$, $N = 200$ with at least 5000 trials in each case.

(iii) Also consider real and complex Wigner matrices with non-Gaussian distribution for the entries.

(iv*) Check numerically whether putting the diagonal equal to zero (in GUE or Wigner) has an effect on the statistics of the largest eigenvalue.

(v*) Bonus: Take a situation where we do not have convergence to the semicircle, e.g., Wigner matrices with Cauchy distribution for the entries. Is there a reasonable guess for the asymptotics of the distribution of the largest eigenvalue?

(vi*) Superbonus: Compare the situation of repelling eigenvalues with "indepentent" eigenvalues. Produce N independent copies $x_1, \ldots, x_N$ of variables distributed according to the semicircle distribution and then take the maximal value $x_{\max}$ of these. Produce a histogram of the statistics of $x_{\max}$. Is there a limit of this for $N \to \infty$? How does one have to scale with N?

Please turn the page.

Problem 29 (10 points)**.** Prove that for all $k \in \mathbb{N}$, the Catalan numbers are bounded by

$$C_k \leq \frac{4^k}{k^{\frac{3}{2}}\sqrt{\pi}},$$

and show that this gives the right asymptotics, i.e., prove that

$$\lim_{k\to\infty} \frac{4^k}{k^{\frac{3}{2}} C_k} = \sqrt{\pi}.$$

Problem 30 (10 points)**.** Work out the details for the "almost sure" part of Corollary 8.8. That is, prove that the largest eigenvalue of GUE(N) converges almost surely to 2 for $N \to \infty$.

Problem 31 (10 points)**.** Let $H_n(x)$ be the Hermite polynomials. The Christoffel-Darboux identity says that

$$\sum_{k=0}^{n-1} \frac{H_k(x)H_k(y)}{k!} = \frac{H_n(x)H_{n-1}(y) - H_{n-1}(x)H_n(y)}{(x-y)\ (n-1)!}.$$

(i) Check this identity for $n = 1$ and $n = 2$.

(ii) Prove the identity for general n.