UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG MATHEMATIK
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Random Matrices

Summer term 2018

Assignment 10

Due: Tuesday, July 3, 2018, before the lecture
Hand in your solution at the beginning of the lecture or drop it into letterbox 47.

Problem 32 (10 points)**.** Consider the rescaled Hermite functions

$$\widetilde{\psi}_N(x) = N^{\frac{1}{12}} \psi_N \left(2\sqrt{N} + xN^{-\frac{1}{6}} \right).$$

(i) Numerically check that the rescaled Hermite functions have a limit for $N \to \infty$ by plotting them for different values of N.

(ii) Familiarize yourself with the Airy function. Compare the above plots of $\widetilde{\psi}_N$ for large N with a plot of the Airy function.

Hint: Use the existing MATLAB implementation of the Airy function:

`https://de.mathworks.com/help/symbolic/airy.html`

Problem 33 (10 points)**.** Read the first section of the following notes:

Per-Olof Persson: Numerical Methods for Random Matrices

(i) Use the MATLAB code from Section 1.2, which relies on the representation via Painlevé II, to produce the Tracy-Widom distribution F_2.

(ii) Compare this F_2 with the histograms you produced for Problem 28.

Please turn the page.

Problem 34 (10 + 5* points)**.** For $N = 100, 500, 1000$ plot in the complex plane the eigenvalues of the $N \times$N random matrix $\frac{1}{\sqrt{N}} A_N$, where all entries of A_N are independent and identically distributed ...

(i) ... Standard Gaussian random variables.

(ii) ... Bernoulli-distributed random variables.

(iii*) ... Cauchy-distributed random variables.

Problem 35 (10 points)**.** Prove that the Hermite functions satisfy the differential equations

$$\psi'_N(x) = -\frac{x}{2}\psi_N(x) + \sqrt{N}\psi_{N-1}(x)$$

and

$$\psi''_N(x) + \left(N + \frac{1}{2} - \frac{1}{4}x^2\right)\psi_N(x) = 0.$$