

Random Matrices

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Winter Semester 2019/20

eigenvalues of a 2×2 -matrix with ± 1 entries

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$$

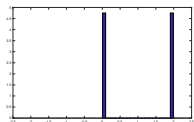
eigenvalues of a 2×2 -matrix with ± 1 entries

$$0, 2 \quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$-2, 0 \quad \begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$$

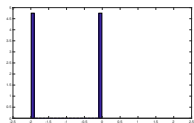
$$-\sqrt{2}, \sqrt{2} \quad \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$$

eigenvalues of a 2×2 -matrix with ± 1 entries



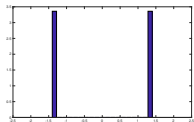
$0, 2$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$



$-2, 0$

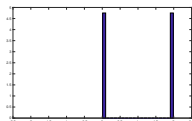
$$\begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$$



$-\sqrt{2}, \sqrt{2}$

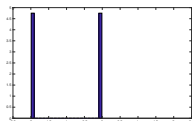
$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$$

eigenvalues of sa 2×2 -matrix with ± 1 entries



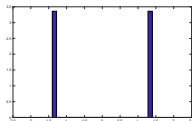
0, 2

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$



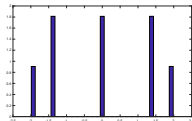
-2, 0

$$\begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$$



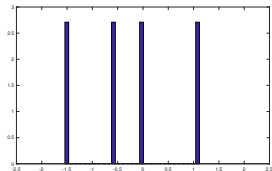
$-\sqrt{2}, \sqrt{2}$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$$

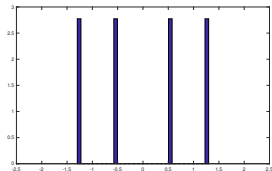


eigenvalues averaged over all $2^3 = 8$ matrices

eigenvalues of a 4×4 -matrix with ± 1 entries

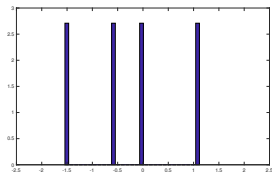


one realization

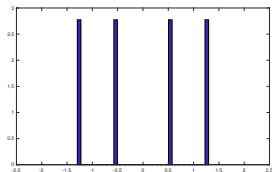


another realization

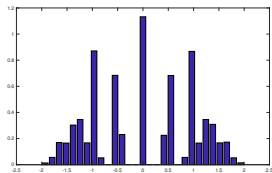
eigenvalues of sa 4×4 -matrix with ± 1 entries



one realization

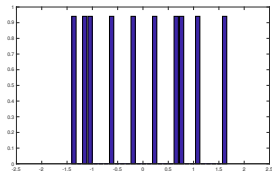


another realization

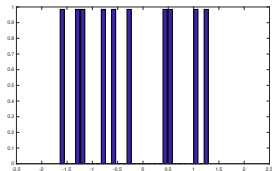


averaged over all $2^{10} = 1024$ matrices

eigenvalues of a 10×10 -matrix with ± 1 entries

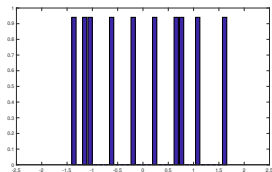


one realization

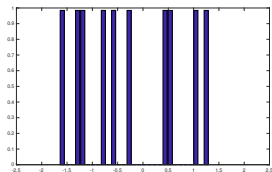


another realization

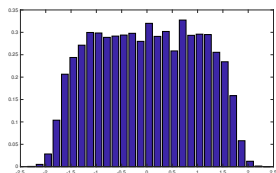
eigenvalues of sa 10×10 -matrix with ± 1 entries



one realization

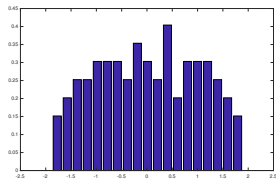


another realization

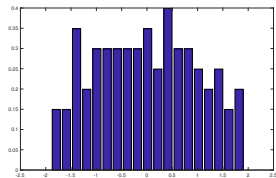


averaged over 50.000 of all 2^{55} matrices

eigenvalues of sa 100×100 -matrix with ± 1 entries

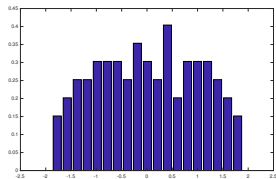


one realization

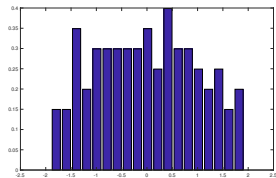


another realization

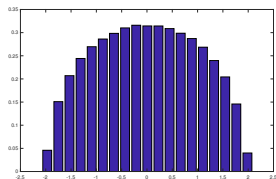
eigenvalues of sa 100×100 -matrix with ± 1 entries



one realization

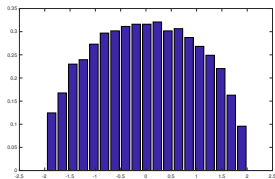


another realization

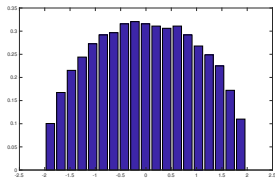


averaged over 1000 of all $2^{50 \times 101}$ matrices

ev of sa 1000×1000 -matrix with ± 1 entries

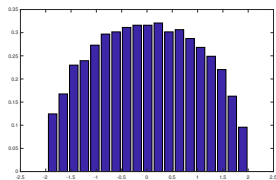


one realization

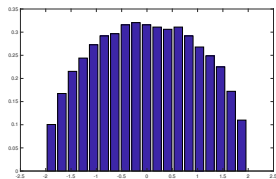


another realization

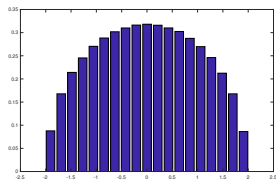
ev of sa 1000×1000 -matrix with ± 1 entries



one realization



another realization



averaged over 100 of all $2^{500 \times 1001}$ matrices