UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG 6.1 – MATHEMATIK
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Assignments for the lecture on
Random Matrices
Winter term 2019/20

Assignment 1
Hand in on Thursday, 24.10.19, Mailbox 040.

Exercise 1 (10 points*).
Make yourself familiar with Mathlab. If you do not have a copy yet, you can get it from here:

`https://unisb.asknet.de/cgi-bin/product/P10011574`

In particular, you should try to generate random matrices and calculate and plot their eigenvalues. If you hand im some histograms similar to the ones shown in the lecture you can earn up to 10 bonus points.
Note: You do not have to hand in this exercise on 24.10.19, you may sumit it at a later date in order to earn the 10 bonus points

Exercise 2 (10 points).
In this problem we want to derive the explicit formula for the Catalan numbers. We define numbers c_k by the recursion

$$c_k = \sum_{l=0}^{k-1} c_l c_{k-l-1} \tag{1}$$

for $k > 0$, with the initial data $c_0 = 1$.

(i) Show that the numbers c_k are uniquely defined by the recursion (1) and its initial data.

(ii) Consider the (generating) function

$$f(z) = \sum_{k=0}^{\infty} c_k z^k$$

and show that the recursion (1) implies the relation

$$f(z) = 1 + z f(z)^2.$$

(iii) Show hat f is a power series representation for

$$z \mapsto \frac{1 - \sqrt{1-4z}}{2z}.$$

Note: You may use the fact that the formal power series f, defined in ii), has a positive radius of convergence.

(iv) Conclude that

$$c_k = C_k = \frac{1}{k+1}\binom{2k}{k}.$$

Exercise 3 (10 points)**.**
Consider the semicircular distribution, given by the density function

$$\frac{1}{2\pi}\sqrt{4-x^2}\,\mathbb{1}_{[-2,2]}, \tag{2}$$

where $\mathbb{1}_{[-2,2]}$ denotes the indicator function of the interval $[-2,2]$. Show that (2) indeed defines a probability measure, i.e.

$$\frac{1}{2\pi}\int_{-2}^{2}\sqrt{4-x^2}\,\mathrm{d}x = 1.$$

Moreover show that the even moments of the measure are given by the Catalan numbers and the odd ones vanish, i.e.

$$\frac{1}{2\pi}\int_{-2}^{2} x^n\sqrt{4-x^2}\,\mathrm{d}x = \begin{cases} 0 & n \text{ is odd} \\ C_k & n = 2k \end{cases}.$$