**UNIVERSITÄT DES SAARLANDES**
**FACHRICHTUNG 6.1 – MATHEMATIK**
Prof. Dr. Roland Speicher
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**Assignments for the lecture on**
*Random Matrices*
Winter term 2019/20

**Assignment 7**
Hand in on Monday, 9.12.19, Mailbox 040.

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**Exercise 1.**
Prove Theorem 7.6 from class: The joint eigenvalue distribution of the eigenvalues of a GUE($N$) is given by a density

$$\hat{c}_N e^{-\frac{N}{2}(\lambda_1^2+\cdots+\lambda_N^2)} \prod_{k<l}(\lambda_l - \lambda_k)^2,$$

restricted on $\lambda_1 < \lambda_2 < \cdots < \lambda_N$, by adapting the proof for the GOE case and parametrizing a unitary matrix in the form $U = e^{-iH}$, where $H$ is a selfajoint matrix.

**Exercise 2.**
In order to get a feeling for the repulsion of the eigenvalues of GOE and GUE compare histograms for the following situations:

- the eigenvalues of an $N \times N$ GUE matrix for one realization
- the eigenvalues of an $N \times N$ GOE matrix for one realization
- $N$ independently chosen realizations of a random variable with semicircular distribution

for a few suitable values of $N$ (for example, take $N = 50$ or $N = 500$).

**Exercise 3.**
For small values of $N$ (like $N = 2, 3, 4, 5, 10$) draw the histogram of averaged versions of GUE($N$) and of GOE($N$) and notice the fine structure in the GUE case. In the next assignment we will compare this with the analytic expression for the GUE($N$)-density from class.