UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG 6.1 – MATHEMATIK
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Assignments for the lecture on
Random Matrices
Winter term 2019/20

Assignment 9
Hand in on Monday, 13.01.20, Mailbox 040.

Exercise 1 (20 points).
Produce histograms for the Tracy-Widom distribution. Plot $(\lambda_{\max} - 2)N^{2/3}$.

(i) Produce histograms for the largest eigenvalue of GUE(N), for $N = 50$, $N = 100$, $N = 200$, with at least 5000 trials in each case.

(ii) Produce histograms for the largest eigenvalue of GOE(N), for $N = 50$, $N = 100$, $N = 200$, with at least 5000 trials in each case.

(iii) Consider also real and complex Wigner matrices with non-Gaussian distribution for the entries.

(iv) Check numerically whether putting the diagonal equal to zero (in GUE or Wigner) has an effect on the statistics of the largest eigenvalue.

(v) Bonus: Take a situation where we do not have convergence to semicircle, e.g., Wigner matrices with Cauchy distribution for the entries. Is there a reasonable guess for the asymptotics of the distribution of the largest eigenvalue?

(vi) Superbonus: Compare the situation of repelling eigenvalues with "independent" eigenvalues. Produce N independent copies $x_1, \dots, x_N$ of variables distributed according to the semicircle distribution and take then the maximal value $x_{\max}$ of these. Produce a histogram of the statistics of $x_{\max}$. Is there a limit of this for $N \to \infty$; how does one have to scale with N?

Exercise 2 (10 points).
Prove the estimate for the Catalan numbers

$$C_k \leq \frac{4^k}{k^{3/2}\sqrt{\pi}} \qquad \forall k \in \mathbb{N}.$$

Show that this gives the right asymptotics, i.e., prove that

$$\lim_{k\to\infty} \frac{4^k}{k^{3/2}C_k} = \sqrt{\pi}.$$

Exercise 3 (10 points).
Let $H_n(x)$ be the Hermite polynomials. The Christoffel-Darboux identity says that

$$\sum_{k=0}^{n-1} \frac{H_k(x)H_k(y)}{k!} = \frac{H_n(x)H_{n-1}(y) - H_{n-1}(x)H_n(y)}{(x-y)\ (n-1)!}.$$

(i) Check this identity for $n = 1$ and $n = 2$.

(ii) Prove the identity for general n.