UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG 6.1 – MATHEMATIK
Prof. Dr. Roland Speicher
M.Sc. Felix Leid

Assignments for the lecture on
Random Matrices
Winter term 2019/20

Assignment 10
Hand in on Monday, 20.01.20, Mailbox 040.

Exercise 1 (10 points).
Work out the details for the "almost sure" part of Corollary 9.8, i.e., prove that almost surely the largest eigenvalue of GUE(N) converges, for $N \to \infty$, to 2.

Exercise 2 (10 points).
Consider the rescaled Hermite functions

$$\tilde{\psi}(x) := N^{\frac{1}{12}} \psi_N(2\sqrt{N} + xN^{-\frac{1}{6}}).$$

(i) Check numerically that the rescaled Hermite functions have a limit for $N \to \infty$ by plotting them for different values of N.

(ii) Familarize yourself with the Airy function. Compare the above plots of $\tilde{\psi}_N$ for large N with a plot of the Airy function.

Hint: MATLAB has an implementation of the Airy function, see

`https://de.mathworks.com/help/symbolic/airy.html`

Exercise 3 (10 points).
Prove that the Hermite functions satisfy the following differential equations:

$$\psi_n'(x) = -\frac{x}{2}\psi_n(x) + \sqrt{n}\psi_{n-1}(x)$$

and

$$\psi_n''(x) + (n + \frac{1}{2} - \frac{x^2}{4})\psi_n(x) = 0.$$